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Sir FRANK DYSON, VICE-PRESIDENT, in the Chair.

Gerald Victor Adendorff, B.Sc., Hilliard's Chambers, Church Square, Cape Town, South Africa;

Benjamin Davies Evans, First Assistant, Royal Observatory, Hong Kong;

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Professor John A. Snyder, Philadelphia Observatory, Broad and Green Streets, Philadelphia, U.S.A.;

William Albert James Weale, B.A., 1 Dyffryn Road, Dyffryn, Pontypridd;

Dr. Harold Wichello, The Crossways, Lache Lane, Chester; and

James Young, A.R.C.Sc., B.Sc. (Lond.), Assistant Lecturer in Physics at Birmingham University, 116 Franklin Road, King's Norton, Birmingham,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Clarence Augustus Chant, Ph.D., Professor of Astrophysics, University of Toronto (proposed by Otto Klotz);

George Selwyn Clark-Maxwell, St. Leonard's Rectory, Bridgenorth (proposed by J. Hargreaves); and
Martin Christopher Johnson, B.A., Physics Department, University of Birmingham (proposed by F. J. M. Stratton).

Sixty presents were announced as having been received since the last meeting, including, amongst others:—

E. N. Nevill, The Lunar Theory: the terms arising from the disturbing action of the Planets.
San Fernando Observatory, Astrographic Catalogue, vol. 5, zone $-6°$.
Sydney Observatory, Astrographic Catalogue, vols. 3 and 4, zones $-51°$ to $-53°$.

The Absorption of Radiation inside a Star. Second Paper.* By A. S. Eddington, M.A., M.Sc., F.R.S.

1. A theoretical investigation has recently been published by H. A. Kramers † which has an important bearing on the problem of absorption and emission of radiation inside a star. His theory leads to formulæ for the intensity of the continuous X-ray spectrum and for the absorption of X-rays by expulsion of K and L electrons, which are in close agreement with laboratory results. At least part of the processes of energy-transfer in the interior of a star are of this nature, and it is desirable to examine how far Kramers' theory elucidates the astronomical problem.

It will be convenient to give first a list of the principal symbols used in this paper, together with the most recent values of the natural constants. The decimal logarithms of the latter are given.

	Logarithm.
H = mass of a hydrogen atom	24·2206
m = mass of an electron = $\text{H}/1845$	$\bar{2}\bar{8}$·9546
e = charge of an electron (electrostatic units)	$\bar{1}\bar{0}$·6789
c = velocity of light	10·4769
$b = \frac{3}{2} \times$ radius of electron $= e^2/mc^2$	$\bar{1}\bar{3}$·4494
u_0 = arithmetic mean speed of electron at 1°	5·7944
R = Boltzmann's constant $= \frac{1}{8}\pi m u_0^2$	$\bar{1}\bar{6}$·1374
a = coefficient of Stefan's law $= (48\pi \text{R}^4/c^3h^3)\sum_1^\infty r^{-4}$	$\bar{1}\bar{5}$·8832
h = Planck's constant	$\bar{2}\bar{7}$·8161
G = constant of gravitation	$\bar{8}$·8235

* First paper, *M.N.*, 83, 32. See also 98, 431.
† *Phil. Mag.*, 46, 836 (November 1923).

Other symbols are, N the atomic number, A the atomic weight, μ the molecular weight, ρ the density, T the temperature, k the mass-coefficient of absorption of material, v the initial velocity of an electron undergoing encounter, ν_0 the frequency corresponding to v ($h\nu_0 = \frac{1}{2}mv^2$), σ the perpendicular from the nucleus on the initial track, so that σv is the angular momentum; M the mass of a star, L the total energy radiated by it per second, $1-\beta$ the ratio of the force of radiation-pressure to gravitation in the interior, η the factor introduced in my former papers depending on the difference of rate of liberation of energy in different parts of the star.

2. When a stream of electrons of velocity v falls on an anticathode a continuous spectrum is emitted, extending from the lowest frequencies observable up to a limit ν_0, such that $h\nu_0 = \frac{1}{2}mv^2$. If n electrons encounter a thin slice of material containing s atoms per sq. cm., the radiation emitted between frequencies ν and $\nu + d\nu$ is found by experiment to be given by the formula

$$C d\nu \,.\, ns\mathrm{N}^2/v^2 \qquad (\nu < \nu_0) \quad . \quad . \quad . \quad (1)$$

where C is a constant independent of the material and of ν.

An electron on capture must emit energy greater than $\frac{1}{2}mv^2$, *i.e.* $> h\nu_0$, and according to the quantum theory the corresponding radiation has a frequency greater than ν_0; hence the radiation (1) is emitted by electrons which are not captured, but lose only part of their energy at the encounter. The electron is switched on to another free track of less total energy. In laboratory conditions the passage is always from greater to less energy with emission of radiation; but in thermodynamical equilibrium (as in the stars) there must be an exactly compensating number of encounters with passage from less to greater energy and absorption of radiation. It is an accepted principle in thermodynamics (though not, I believe, rigorously demonstrated) that in equilibrium the various modes of energy-transfer must balance independently of one another, each process being capable of direct reversal.

The above process is occurring in the stars, and I shall examine to what extent it contributes to the stellar absorption-coefficient. I base the discussion on Kramers' theory, but it would be equally possible to base it on the empirical law (1) with which the theory agrees. The theory, however, provides the justification for extrapolating beyond the range of experiment.

Kramers first calculates the amount of energy of frequency ν to $\nu + d\nu$ radiated per encounter according to Larmor's classical radiation formula. The result is *

$$\frac{4\pi^2\mathrm{N}^2e^6}{c^3m^2\sigma^2v^2}\mathrm{P}(\gamma)d\nu \,. \quad . \quad . \quad . \quad . \quad (2)$$

where

$$\gamma = 2\pi\nu\sigma^3v^3m^2/\mathrm{N}^2e^4 \quad . \quad . \quad . \quad . \quad (3)$$

and P is a certain function having the property

$$\int_0^\infty \frac{\mathrm{P}(\gamma)}{\gamma}d\gamma = \frac{4}{\pi\sqrt{3}} \quad . \quad . \quad . \quad . \quad (4)$$

* *Loc. cit.*, equation (50).

He assumes that up to the limit ν_0 this result will still hold good *statistically* on the quantum theory.

For n electrons encountering a thin sheet containing s atoms per sq. cm., the number of encounters between σ and $\sigma + d\sigma$ is $ns \cdot 2\pi\sigma d\sigma$. Hence by (2) the energy-emission is

$$nsdv \cdot \frac{8\pi^3 N^2 e^6}{c^3 m^2 v^2} \int_0^\infty \frac{P(\gamma)}{\sigma} d\sigma.$$

But by (3), $d\gamma/\gamma = 3d\sigma/\sigma$; hence by (4) the integral is $4/3\pi\sqrt{3}$. Accordingly the emission is

$$nsdv \cdot \frac{32\pi^2}{3\sqrt{3}} \cdot \frac{N^2 e^6}{c^3 m^2 v^2} \quad . \quad . \quad . \quad . \quad (5)$$

agreeing with the empirical result (1). The experimental constant C is also in close agreement.

Consider a cubic centimetre of stellar material. The number of atoms is ρ/A_H, and the number of atoms + free electrons is ρ/μ_H. If $1/f$ is the number of free electrons per atom, the number of electrons per cu. cm. is $\rho/\mu_H(1+f)$. In stellar conditions f is usually negligible, but we may as well retain the factor. Accordingly, for the emission per cu. cm. per second, we write in (5)

$$s = \frac{\rho}{A_H} \qquad n = \frac{\rho v}{\mu_H(1+f)} \qquad \int_0^{\nu_0} dv = \nu_0 = \tfrac{1}{2}mv^2/h,$$

giving

$$\frac{16\pi^2}{3\sqrt{3}} \cdot \frac{\rho^2}{H^2 A \mu(1+f)} \cdot \frac{N^2 e^6 v}{c^3 mh} \cdot \quad . \quad . \quad . \quad (6)$$

If v is different for different electrons we must evidently take the arithmetic mean, so that $v = u_0 T^{\frac{1}{2}}$. In radiative equilibrium the emission (6) may be equated to the absorption which is $k\rho a c T^4$. Hence we have

$$\mu(1+f)k\frac{T^{\frac{1}{2}}}{\rho} = \frac{16\pi^2}{3\sqrt{3}} \frac{N^2}{A} \frac{e^6 u_0}{H^2 a c^4 mh}$$

$$= \frac{128\pi}{3\sqrt{3}} \cdot \frac{N^2}{A} \cdot \frac{Rb^2 c}{H^2 a u_0} \cdot \frac{e^2}{hc} \cdot \quad . \quad . \quad . \quad (7)$$

using the values of R and b given in § 1.

My theory of nuclear capture (including the "relativity correction") gave the formula,*

$$\mu(1+f)k\frac{T^{\frac{1}{2}}}{\rho} = 10{\cdot}8 \frac{N^2}{A} \cdot \frac{Rb^2 c}{H^2 a u_0} \quad . \quad . \quad . \quad (8)$$

If k_x is the absorption-coefficient corresponding to the continuous

* *M.N.*, 83, 43, equation (16); *ibid.* 103. I here neglect the nuclear radius d in comparison with $\frac{1}{2}Nb$, and reinsert the correcting factor $(1+f)$; u is replaced by $\frac{1}{4}\pi u_0$.

X-ray spectrum, and k_n the coefficient corresponding to my theory of nuclear capture, we have by (7) and (8)

$$\frac{k_n}{k_x} = \frac{hc}{7{\cdot}17e^2} = 120{\cdot}2 \quad . \quad . \quad . \quad . \quad (9)$$

the ratio being independent of temperature, density, ionisation, and chemical constitution. If I am right in my former conclusion that k_n agrees closely with astronomical observation, it follows that k_x accounts for only about $\frac{1}{100}$ of the whole absorption in the stars.

3. We turn next to encounters which result in capture of the electron. The energy to be emitted is then greater than $h\nu_0$ and its frequency is therefore greater than ν_0. According to classical electrodynamics (5) is still valid for $\nu > \nu_0$; but, according to the quantum theory, this continuous spectrum must be replaced by a series of lines corresponding to the series of possible permanent orbits of the captured electron. In contrast to the continuous series of possible free orbits there exists only a limited number of possible captured orbits. Kramers postulates that this replacement is of the simplest possible kind, viz. that as far as the limit of the line spectrum each element of the continuous spectrum is transferred to the nearest line. Thus, for a considerable range beyond ν_0 the energy given by (5) is actually emitted, although not in the continuous distribution of frequency which the classical theory predicts. The formula (5) is cut off finally at some value of ν beyond the limit of the line spectrum.

The line of highest frequency corresponds to capture in a K orbit. Let the negative energy of this orbit be $h\nu_K$, so that the frequency of the line is $\nu_0 + \nu_K$. For reasons explained in his paper, Kramers takes $\nu_0 + 4\nu_K$ to be the dividing limit, which separates the energy gathered towards this line from the energy swept away from it and therefore unable to contribute to the line spectrum. Thus $\nu_0 + 4\nu_K$ is the ultimate limit of (5). Actually the greater part of the classical radiation is above this limit, so that the whole emission on Kramers' theory is very much less than on the classical theory. I have shown elsewhere that the classical theory would give much too high an absorption-coefficient in the stars,* so that the modification is in the right direction according to astronomical evidence.

Capture in a K orbit can only occur if a K orbit is vacant. If the atom has not been ionised so far as to lose either of its K electrons the line of highest frequency corresponds to an L orbit, and the emission (5) is then cut off about midway between $\nu_0 + \nu_L$ and $\nu_0 + \nu_K$. Similarly, if the lowest vacant orbit is an M orbit, the limit is about midway between $\nu_0 + \nu_M$ and $\nu_0 + \nu_L$. The limit thus shifts according to the degree of ionisation.

Although the theory is not very explicit on the point, we shall assume that each individual orbit has a definite partition of the classical energy associated with it, the emission of this energy only occurring if the orbit is vacant. That is to say, if one of the K orbits is occupied the K line is of half strength; and the intensity of the L line decreases in steps of $\frac{1}{8}$ for each L orbit filled. (The normal complement is two K electrons and eight L electrons.)

* *M.N.*, **83**, 44.

From the theoretical standpoint Kramers' postulate can only be regarded as an ingenious conjecture; but the important fact is, that coupled with a thermodynamical argument* it leads to formulæ for the absorption of X-rays in approximate agreement with laboratory experiment. As in § 2 we may regard his theory as a convenient means of applying and extrapolating laboratory results in the stellar problem.

Let $\nu_0+\nu_1$ be the limiting frequency, where ν_1 depends on the state of ionisation of the atom according to the foregoing explanation. Then the classical spectrum from 0 to ν_0 is associated with the continuous emission, and from ν_0 to $\nu_0+\nu_1$ with the line-emission. Hence by (5) the line-emission (by capture) is to the continuous emission in the ratio $\nu_1 : \nu_0$; so that if k_c is the absorption-coefficient corresponding to capture (and its converse ionisation)

$$k_c/k_x = \nu_1/\nu_0.$$

Taking account of the unequal velocities of electrons distributed according to Maxwell's law, the ratio is that of the mean values of

$$h\nu_1/v : \tfrac{1}{2}mv^2/v,$$

which reduces to

$$\frac{k_c}{k_x} = \frac{h\nu_1}{\mathrm{RT}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

To determine ν_1 we first simplify the problem by considering an atom with a regular succession of energy-levels at short intervals, one electron to each level. Let x be the fraction of atoms ionised as to the level $h\nu$, then †

$$\log\frac{x}{1-x}+\frac{h\nu}{\mathrm{RT}} = \log\left\{\frac{(2\pi m)^{\frac{3}{2}}\mathrm{R}^{\frac{5}{2}}}{h^3}\cdot\frac{\mathrm{T}^{\frac{5}{2}}}{\mathrm{P}_e}\cdot\frac{s}{b(\mathrm{T})}\right\} \quad . \quad . \quad (11)$$

Here P_e is the electron-pressure (practically the whole material pressure), s the symmetry number for the electron group considered, and $b(\mathrm{T})$ a correcting factor unimportant in stellar conditions.

Consider the value of ν for which $x=\frac{1}{2}$; the ionised half of the atoms will then have a limit ν_1 lying about midway between ν and the next lower level; the unionised half will have a limit ν_1 lying about midway between ν and the next higher level. Hence in the mean this value of ν is a suitable approximation to ν_1. Accordingly, inserting the value $\mathrm{P}_e = \mathrm{R}\rho\mathrm{T}/\mathrm{H}\mu$, and setting $\sigma=1$ in accordance with our simplifying assumption

$$\frac{h\nu_1}{\mathrm{RT}} = \log\left\{\frac{(2\pi m\mathrm{R})^{\frac{3}{2}}\mathrm{H}}{h^3}\cdot\frac{\mu\mathrm{T}^{\frac{3}{2}}}{\rho}\right\} \quad . \quad . \quad . \quad (12)$$

* The argument in Kramers' paper does not seem to me convincing, but the formula to which it leads is obtainable in a rigorous manner (Milne, *Phil. Mag.*, **47**, 209).

† See Fowler and Milne, *M.N.*, 83, 407, equation (1), or, more strictly, equation (8). Actually in a star there is very little overlapping of successive stages of ionisation, though the overlapping might be more important if the atom had the simplified structure here assumed.

The right-hand side will vary comparatively slowly over a considerable range of temperature, so that by (10) the ratio k_c/k_x is fairly constant.

In § 6 we shall examine more fully the results for actual atoms, taking account of the grouping of the electron orbits. In the main these agree well with the results for the somewhat idealised atom here considered.

4. The best astronomical determination of the absorption-coefficient is obtained from the binary star Capella. This has well-determined visual (interferometer) and spectroscopic orbits, so that the parallax is known with greater accuracy than for any other giant binary star.* The following data for the brighter component are adopted—

$$\text{Mass} = 4{\cdot}2\odot\ ;\ \text{abs. mag.} = -0{\cdot}3\ ;\ \text{eff. temp.} = 5500^\circ\ ;\ \mu = 2{\cdot}2.$$

This molecular weight, which is lower than I have previously considered probable, is justified in § 6.

Adopting $+5{\cdot}0$ for the sun's absolute magnitude, we find L/M for Capella 31·4 times greater than for the sun, whence

$$L/M = 60{\cdot}0.$$

The principal formulæ of the radiative theory of stellar equilibrium are

$$1-\beta = 0{\cdot}00260\ \beta^4\mu^4(M/\odot)^2 \quad . \quad . \quad . \quad (13)\dagger$$

$$L = 4\pi cGM(1-\beta)/k\eta \quad . \quad . \quad . \quad . \quad (14)$$

which give

$$1-\beta = {\cdot}2833 \qquad \eta k = 118{\cdot}6.$$

At the centre of the star we may take $\eta = 2{\cdot}5$.‡ Hence the astronomical value of the absorption-coefficient at the centre of Capella is

$$k_a = 47{\cdot}5.$$

From the absolute magnitude and effective temperature we at once determine the surface area and hence the radius and mean density. The mean density is ·00260. Then by the theory of radiative equilibrium the central density is 0·1414. Also

$$\frac{T^3}{\rho} = \frac{3R(1-\beta)}{H\mu a\beta} \quad . \quad . \quad . \quad . \quad (15)$$

giving the central temperature $9{\cdot}56.10^6$ degrees. Inserting these values

* P. W. Merrill, *Ap. J.*, **56**, 40.

† The computations for Capella were made before the final revision of the table of natural constants in § 1. The coefficient ·00260 depends on certain calculations by Emden. The values of the mass of the sun and of the gas-constant (R/H) used by him were not very accurate. With modern data I find that the coefficient should be ·00308. The change is very trifling, and I have not thought it worth while to alter the computations in this paper which were made with the older data. It is compensated if we reduce the molecular weight for Capella from 2·2 to 2·1.

‡ See *M.N.*, **83**, 40. If ηk is not quite constant the above value will be an average and not exactly the value at the centre. In deciding on the factor 2·5 we have regard to the law of variation of k as well as to the probable variation of η.

of T and ρ in (7) and taking the material to be iron (N = 26, A = 56), the result is

$$k_x = 0{\cdot}65.$$

Thus the ratio k_x/k_a is ·0138, agreeing with our previous statement that k_x is only about a hundredth part of the actual absorption.

By (9) the value according to my theory of nuclear capture is

$$k_n = 78{\cdot}4,$$

agreeing well enough with k_a.

By combining (7), (14), and (15) we obtain the convenient comparison

$$\frac{k_x}{k_a} = \frac{32}{9\sqrt{3}} \cdot \frac{b^2e^2}{\mathrm{G}\mathrm{H}u_0hc} \cdot \frac{\mathrm{N}^2}{\mathrm{A}} \cdot \frac{\mathrm{L}}{\mathrm{M}} \cdot \frac{\beta}{(1-\beta)^2} \cdot \frac{\eta}{\mathrm{T}^4} \cdot \frac{1}{(1+f)} \quad . \quad (16)$$

By this we can examine whether any legitimate changes in the adopted data would bring k_x and k_a closer together. The astronomical quantity L/M can scarcely be much in error. The effective temperature of Capella is only required in determining T^4, and k_x/k_a will vary inversely as the effective temperature; there is clearly not much room for alteration on this account. The value of $1-\beta$ has been most doubtful in previous investigations owing to its rapid variation for small changes of μ; but I now feel satisfied that μ must be close to 2·2, unless the star contains a rather large proportion of hydrogen. A higher value of μ would widen the difference between k_x and k_a. The factor N^2/A is very nearly proportional to N, and if the star were made of exceptionally heavy elements might be increased twofold or, conceivably, threefold; but for heavy elements μ would be somewhat larger than 2·2 and the gain would be partly neutralised. In order to double η the generation of sub-atomic energy must be concentrated to the centre of the star to an extravagant extent; moreover this local increase of η is not fully effective since our formula (16) assumes that ηk at the centre is equal to its average value in the star. Finally, the law of dependence of k_x on T and ρ is the same as that of k_n; and we shall find that the dependence of k_c is nearly the same. I have shown * that this law makes the assumption $\eta k = \text{const.}$ of the radiative theory an exceptionally close approximation, and it seems impossible that any important error should have arisen from failure of the theoretical approximations.

It seems then that the ratio $k_x/k_a = {\cdot}0138$ is entitled to considerable confidence. There seems to be no chance of an appreciable increase unless the star contains an unduly large proportion of hydrogen.

5. Inserting in (12) the values found for the centre of Capella, viz.

$$\mathrm{T} = 9{\cdot}56.10^6 \qquad \rho = 0{\cdot}1414,$$

we obtain

$$h\nu_1/\mathrm{RT} = 7{\cdot}505 \quad . \quad . \quad . \quad . \quad (17)$$

Hence by (10)

$$k_c = 7{\cdot}505\ k_x = 4{\cdot}91,$$

* *M.N.*, **83**, 32, § 5.

and the total theoretical absorption is

$$k_c + k_x = 5{\cdot}56,$$

which is still only an eighth of the astronomical value.

By (17) we find that the limit ν_1 corresponds to a wave-length

$$\lambda_1 = 2{\cdot}03\text{Å}.$$

For elements Nos. 24–57 this falls between the K and L limits so that the ionisation is at a stage between the removal of the eight L electrons and of the two K electrons. We may consider in particular iron for which the limits are

$$\lambda_K = 1{\cdot}74\text{Å} \qquad \lambda_L = 12\text{Å},$$

the latter being a very rough value. λ_1 is so far past the L limit that the L ionisation must be practically complete; the K ionisation is beginning. For removal of one K electron the symmetry number $s = 2$ should be replaced in (12), so that λ_1 is reduced to 1·88. As this is very near to λ_K, we may expect about half the atoms to have lost a K electron. Actually by (11) we find $x = \frac{1}{3}$. For a second ionisation λ_1 reverts to 2·03 and λ_K drops to 1·35 (the theoretical value for a nucleus attended by one electron). Only about $\frac{1}{40}$ of the atoms which have lost one K electron lose the second, so that double ionisation is negligible.

About $\frac{2}{3}$ of k_c was due to the range 2·03 to 6Å corresponding to K captures. Our more detailed discussion indicates that the range should be taken from $\frac{1}{4} \times 1{\cdot}35$ to 6, and the result divided by 6, because only $\frac{1}{6}$ of the K orbits are unoccupied. This will increase the K portion of the absorption by 44 per cent. or the whole coefficient k_c by 29 per cent. The revised value is $k_x + k_c = 7{\cdot}0$ – an unimportant change compared with 5·56 for the idealised atom.

Individual elements at particular parts of the star will give results somewhat above or below the smooth formula (12). We shall undertake a more extensive check on its approximate validity in § 6.

It is characteristic of stellar conditions that $h\nu_1/\text{RT}$ is rather large. The effect is to reduce the overlapping of successive stages of ionisation, and to make the proportion of electrons excited into higher quantum orbits small.

The centre of Capella corresponds to one extreme of the conditions in the star. To obtain a fair idea of the general conditions we ought also to consider the other extreme. We take a point on the surface of a sphere enclosing $\frac{9}{10}$ of the mass. The temperature here is lower in the ratio $1/3{\cdot}6$ and the density in the ratio $(1/3{\cdot}6)^3$. Hence by (7) k_x is increased in the ratio $(3{\cdot}6)^{\frac{1}{2}}$; and by (12) $h\nu_1/\text{RT}$ is increased by $\frac{3}{2}\log 3{\cdot}6$. Thus

$$k_x = 1{\cdot}23 \qquad h\nu_1/\text{RT} = 9{\cdot}426 \qquad k_c + k_x = 12{\cdot}9.$$

The increase of absorption from 5·56 to 12·9 would nearly compensate the drop of η from 2·5 to 1 and so justify the assumption $\eta k = \text{const.}$ used in the astronomical calculations.

Further, we have

$$\lambda_1 = 5{\cdot}8\text{Å},$$

so that iron and neighbouring elements are still fully ionised as to the L electrons.

6. It is found by experiment that for homogeneous radiation of frequency above the K limit the absorption-coefficient per atom per sq. cm. is $CN^4\lambda^3$ where $C = 0{\cdot}0236$. About $\frac{1}{7}$ of this persists below the K limit and must be attributed to L electrons; hence for absorption by K electrons we may take $C = 0{\cdot}020$. Kramers' theory gives the same law and verifies C to within a factor 2. The absorption-coefficient per gm. per sq. cm. is thus:—

$$CN^4\lambda^3/A_H \qquad C = 0{\cdot}020,\ \lambda < \lambda_K \quad . \quad . \quad . \quad (18)$$

Hence, for black-body radiation at temperature T, the absorption-coefficient is

$$\left.\begin{aligned} k_K &= \frac{CN^4}{A_H}\left(\int_{\nu_K}^{\infty}\frac{\lambda^3\nu^3 d\nu}{e^{h\nu/RT}-1} \div \int_0^{\infty}\frac{\nu^3 d\nu}{e^{h\nu/RT}-1}\right) \\ &= \frac{CN^4}{A_H}\left(\frac{ch}{RT}\right)^3 \frac{1}{6a}\log\frac{1}{1-e^{-h\nu_K/RT}} \end{aligned}\right\} \quad . \quad . \quad (19)$$

where $a = 1^{-4} + 2^{-4} + 3^{-4} + \ldots = 1{\cdot}0823$.

The coefficient k_L due to absorption by L electrons is given by a similar formula, but with a coefficient $C' = \frac{1}{6}C$ (roughly).

The coefficients calculated by Kramers' theory earlier in this paper were strictly emission-coefficients, which were equated to the absorption-coefficients owing to thermodynamical equilibrium. We shall now require the emission-coefficients k_K', k_L' for various elements in order to compare with the absorption-coefficients k_K and k_L calculated by (19). For iron the range $\lambda = 2{\cdot}03$ to ∞ gave a coefficient $4{\cdot}91$; we have merely to alter this in proportion to the range of frequencies associated by Kramers' postulate with K capture and L capture respectively. These ranges are not very accurately defined, but we may take them as roughly $4\nu_K$ to $\frac{1}{2}\nu_K$ and $\frac{1}{2}\nu_K$ to $\frac{1}{2}\nu_L$. For elements other than iron the result must be multiplied proportionately to N^2/A (equation (7)).

For iron at the centre of Capella, we find in this way

$$k_K = 8{\cdot}3 \qquad k_K' = 20{\cdot}0.$$

Here, of course, k_K refers to a neutral atom, being derived from the results of laboratory experiment. Presumably it will not be much disturbed by ionisation, provided that the K electrons are retained; but if any K electrons are lost it must be proportionately reduced. Thus, if x is the proportion of K electrons ionised, the coefficient is reduced to $k_K'' = (1-x)k_K$.

Similarly k_K' is the theoretical value for fully ionised atoms. If any of the K orbits are occupied it must be reduced proportionately. Thus, for incomplete ionisation the emission-coefficient is xk_K'.

This gives us a vivid picture of the process of ionisation. If $(1-x)k_K > xk_K'$, more absorption than emission occurs and more K electrons are expelled than captured. Thus the ionisation x increases

until the inequality disappears. In equilibrium the ionisation x and the actual coefficient k_K'' both of absorption and emission are determined by

$$k_K'' = (1 - x)k_K = xk_K'.$$

With the above values for iron, $x = 29$ per cent., $k_K'' = 5{\cdot}9$.

By general thermodynamical principles this balance must occur independently of other processes of energy-transfer in the material.

By this method we can trace in detail the behaviour of a representative selection of the elements. The following table shows how out of the combination of the laws of absorption, emission, and ionisation a fairly regular absorption-coefficient emerges in the last column. The adopted values of ν_K and ν_L were taken from experimental data, the latter being somewhat rough because the L level is really composite. I have made no allowance for the gradual shift of level, as successive electrons of the group are removed; this no doubt produces some inaccuracy.

Values at Centre of Capella.

Element.	k_K.	k_K'.	x.	k_K''.	k_L.	k_L'.	x.	k_L''.	$k_K''+k_L''+$.
Al (13)	1103	2·3	·998	2·3	...	...	...	...	2·3
A (18)	336	6·1	·98	6·0	...	...	. .	...	6·0
Ti (22)	70·6	11·7	·86	10·0	...	...	...	...	10·0
Fe (26)	8·3	20·0	·29	5·9	2950	2·5	·999	2·5	8·4
As (33)	·05	40	·001	·05	2240	4·9	·998	4·9	5·0
Zr (40)	...	...	...	...	1090	9·1	·991	9·0	9·0
Ag (47)	...	...	...	...	366	14·9	·96	14·3	14·3
Cs (55)	...	...	...	...	65	23·4	·74	17·3	17·3
Sm (62)	...	...	...	...	10·0	34·4	·23	7·7	7·7+
Er (68)	...	...	...	...	1·54	46·3	·03	1·5	1·5+

Beyond atomic number 60 the absorption by M electrons begins to be appreciable so that the total absorption, which appears to be dropping for the last two elements, is sustained. The data are insufficient for a detailed calculation.

In § 5 we found $k_c = 4{\cdot}9$ for iron, increasing proportionately to the atomic number. The results in the last column of the table suggest a slight increase *—say about 50 per cent.; but on the whole there is very fair accordance between the two methods. Of course, the large coefficient for Ti and Cs would not render a star constructed of these elements in any way exceptional; it only means that the conditions at the *centre* of Capella are critical for these two elements. For other points in the star the wave in the last column shifts up or down among the elements, and it should be smoothed out before a general application of the results is attempted.

Using the values of the ionisation x determined in the foregoing table, we can calculate the mean molecular weight at the centre of

* This is probably due to the fact that the experimental value of C is somewhat larger than that predicted by Kramers' theory.

Capella on different hypotheses as to the elements constituting it. The results are :—

C.	O.	Al.	A.	Ti.	Fe.	As.	Zr.	Ag.	Cs	Sm.	Er.
$\mu=1{\cdot}7$	1·8	1·9	2·1	2·1	2·2	2·3	2·3	2·3	2·6	2·7	2·8

The molecular weight will increase very slightly as we go away from the centre, and the mean value (to be used in our formulæ) corresponds to a point where the temperature is $\frac{2}{3}$ of the central temperature.* We may probably assume that elements above Ag are not unduly abundant; allowing a reasonable proportion of lighter elements such as C, N, and O, it would seem that $\mu=2{\cdot}2$ ought to be very nearly correct. Of course this value should not be applied indiscriminately to all stars without considering their actual temperatures, but it is probably typical of the majority.

The lowered value of the molecular weight is due to our conclusion that in all except the heaviest atoms the L electrons are ionised, whereas the pioneer investigation of Eggert seemed to show that the L ring was intact. Milne (*Phil. Mag.*, 47, 209) was the first to show that the L ionisation occurs.

7. Hydrogen is an exceptional element and gives results differing to an extraordinary degree from every other substance. In the following table we compare the results obtained for it with those for the next element (helium) as well as for a mean element (iron). Hydrogen and helium will be fully ionised in Capella, and owing to the low value of ν_K, k_c is negligible compared with k_x; the calculation is thus especially simple. Owing to γ_0 being less than 0·2 (see § 8) k_x should be a little higher for H and He than the values here given.†

Values for Centre of Capella.

	Hydrogen.	Helium.	Iron.	9 Fe ; 1 H.	8 Fe ; 2 H.
k_x	7·0	0·16	0·65	1·09	1·90
k_x+k_c	7·0	0·16	5·56	9·21	15·9
k_a	0·47	16·2	47·5	28·2	15·8
$1-\beta$	·00283	·0966	·283	·168	·0940
μ	0·5	1·33	2·2	1·66	1·32
$10^{-6}T$	3·02	7·30	9·56	8·39	7·25

The discrepancy between calculated absorption and observed absorption, which is $\frac{1}{8}$ for iron, becomes worse for lighter elements until it reaches $\frac{1}{100}$ for helium; then for hydrogen it is suddenly reversed to 15. Evidently agreement could be secured by taking a suitable mixture of hydrogen with heavier elements. The last two columns show the results

* This is the result of actual calculation. There is no difficulty in introducing varying molecular weight into the equations of stellar equilibrium.

† In these calculations Capella is supposed to be constituted wholly of the element under consideration; the appropriate molecular weight is used and the central temperature calculated accordingly. The table in § 5 compared the coefficients for various elements under uniform conditions of temperature and pressure corresponding to molecular weight 2·2.

if Capella contains 10 per cent. and 20 per cent. by weight of hydrogen; in the latter case the agreement is exact.

I think it is scarcely possible to admit so large a proportion of hydrogen (14 hydrogen atoms to each iron atom); the correspondingly low value of $1-\beta$ is difficult to admit.* But some hydrogen must be present, and the quantity may be sufficient to reduce the discrepancy between theory and observation to a factor less formidable than 8. It must be emphasised that no element other than hydrogen has this effect.

It will be noticed that hydrogen and a heavy element mixed together produce greater absorption (k_x+k_e) than either of them singly. By a cunning division of labour the hydrogen provides the free electrons for the iron atoms to capture.

8. It is stated by Kramers that his formulæ are limited by two conditions. The electrons providing the radiation must have tracks such that—

(1) The classical radiation is small compared with the initial energy.
(2) The eccentricity is nearly unity.

The first condition is rather mysterious and we accept it on Kramers' authority. Apparently he anticipates a breakdown of the correspondence of classical and quantum theory when it is not satisfied. The second condition arises from the mathematical approximation; a factor (denoted by g' in the original paper) having been taken to be unity.

If τ is the ratio of the classical radiation to the initial energy, we have (*M.N.*, 83, 43 (18), *et seq.*)

$$\tau=\frac{4\pi N^4 b^5 e^7}{\sigma^5 v^7}.$$

Hence eliminating σ by means of (3)

$$\tau=\frac{2(2\pi)^{\frac{3}{2}}e^{\frac{1}{2}}b^{\frac{5}{2}}}{\gamma^{\frac{5}{2}}}\cdot\frac{N^{\frac{3}{2}}\nu^{\frac{5}{2}}}{v^2} \quad . \quad . \quad . \quad . \quad (20)$$

It is true that we have to consider velocities for which v^2 is 10 to 20 times lower than in the experiments which check Kramers' theory; but on the other hand the verification extends to heavy elements and to frequencies at least 10 times greater than our limit ν at the centre of Capella. Thus the value of τ in our problem falls well within the range for which the theory has been verified, and our application of the theory is justified.†

According to Kramers the second condition is well satisfied if the value γ_0 of γ for a track which is a rectangular hyperbola is not less

* The calculated brightness of a giant star of mass equal to the sun, containing 20 per cent. of hydrogen, is impossibly low.

† For iron at the centre of Capella and frequency at the limit ν_1, (20) gives $\tau=\cdot0046\,\gamma^{-\frac{5}{2}}$. A graph of $P(\gamma)/\gamma$, showing the relative contribution of different values of γ to the total emission, is given by Kramers, *loc. cit.*, p. 858. It appears that the range $\gamma<0\cdot1$ is unimportant.

than 0·2. If $\gamma_0 < 0{\cdot}2$ the formula must be multiplied by a slowly-increasing factor which only reaches 2 when $\gamma_0 = 0{\cdot}02$. We have

$$\gamma_0 = \frac{2\pi N b}{\lambda}\frac{c^3}{v^3}.$$

At the centre of Capella c/v is about 15. Taking $N = 20$ we find that the condition is satisfied for radiation of wave-length less than 60Å—practically the whole. Evidently the increase of absorption from this cause is very trifling.

We have also to consider a phenomenon called by Einstein "negative Einstrahlung." Ostensibly Kramers' formula gives the emission by encounters of electrons with atoms when there is no external field of radiation. When a field of radiation in equilibrium with the electronic velocities is present the emission is stimulated, and according to thermodynamics it is increased in the ratio $e^y/(e^y - 1)$, where $y = h\nu/\mathrm{RT}$.*

The result of applying this correction to Kramers' formula would be to make the emission infinite. The integral of $d\nu.e^y/(e^y - 1)$ is

$$\nu + \mathrm{RT}\log(1 - e^{-y})/h,$$

and this must replace ν in the integration of (5). Taken between a large limit ν_1 and a small limit ν_0, this increases the emission approximately in the ratio

$$1 + \frac{1}{y_1}\log\frac{1}{y_0}.$$

With $y_1 = 8{\cdot}5$ (in accordance with (17)) extremely small values of y_0 are required to give any substantial increase. Thus, to double the emission we must take ν_0 in the infra-red region, and to triple the emission we must include Hertzian waves. Of course this enormous stimulation of long wave-length radiation is physically absurd, and we must examine more carefully the question of applicability of Einstein's correction.

The average classical emission of radiation between ν and $\nu + d\nu$ at one encounter was found to be equal to $C d\nu$, where C is independent of ν. This is interpreted on the quantum theory as signifying that the probability of emission of a quantum at the encounter is $C d\nu/h\nu$. This increases as ν diminishes, so that we are led to introduce probabilities greater than unity. A probability 10 would no doubt signify that the electron undergoes 10 successive switches during a single encounter; but in that case our calculation of the probability of the 10th switch is systematically in error, because we have based it on the initial track of the electron whereas the 9 previous switches have each reduced the energy.

The rigour of our calculation is restored if we can arrange the conditions so that the probability of the later switches is precisely the same as that of the earlier switches, *i.e.* so that the energies of the electrons are not systematically decreased during the progress of the encounters. This is precisely the condition of thermodynamical equilibrium in a field of radiation; for then an equal number of back-switches occur in the

* See Milne, *Phil. Mag.*, 47, 222.

course of the encounters, and it is a matter of accident whether the energy of the track at any moment is greater or less than the average.

I think that this throws light on Einstein's mysterious result. The stimulation of emission is indirect and due to a continual restoration of conditions favourable to emission preventing the saturation effect which would occur in the absence of radiation. In our calculation the saturation effect has been ignored, so that it is illegitimate to remove it a second time by applying Einstein's correction.

9. We have reached the result that the process of emission and absorption considered by Kramers accounts for only about $\frac{1}{8}$ of the absorption in the stars, and the discussion in § 8 does not reveal any neglected theoretical consideration which could appreciably raise the proportion. In this and the following section we shall consider two other processes of energy-transfer in order to ascertain whether they help to make up the deficit. The results prove to be negative.

We shall calculate the emission of radiation due to the encounters of the free electrons with one another. It may be anticipated that for these encounters the classical radiation formula will be statistically correct, since there is now no question of capture, and the emission of *any* frequency switches the electron into a possible orbit. It seems to be a natural consequence of Kramers' view of the origin of the classical radiation that (apart from special modifications when the switch involves capture) there should be no essential dissimilarity between the effects of repulsive and attractive accelerations.

We refer the orbit of the radiating electron and the quantities σ, v, etc., to the centre of mass of the two encountering electrons. The repulsion is equal to an acceleration from this centre of mass of amount α/r^2, where $\alpha = e^2/4m$. Let the orbit be the hyperbola

$$\frac{l}{r} = \epsilon \cos\theta - 1.$$

If q is the constant of areas

$$\sigma v = r^2 d\theta/dt = q \qquad q^2 = \alpha l \quad . \quad . \quad . \quad (21)$$

Also since σ is equal to the minor axis of the hyperbola

$$\epsilon^2 - 1 = l^2/\sigma^2 = \sigma^2 v^4/\alpha^2 \quad . \quad . \quad . \quad . \quad (22)$$

Let $\epsilon = \sec\phi$, so that 2ϕ is the angle between the asymptotes, and

$$\tan\phi = \sigma v^2/\alpha \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The classical radiation is

$$\frac{2}{3}\frac{e^2}{c^3}\left(\frac{\alpha}{r^2}\right)^2 dt = \frac{2}{3}\frac{e^2\alpha^2}{c^3 l^2 q}(\epsilon\cos\theta - 1)^2 d\theta$$

$$= \frac{2}{3}\frac{e^2\alpha^4}{c^3 q^5}(\sec\phi\cos\theta - 1)^2 d\theta.$$

Integrating between $-\phi$ and $+\phi$, the whole radiation at the encounter is

$$\frac{2}{3}\frac{e^2a^4}{c^3\sigma^5v^5}\{\phi(\sec^2\phi+2)-3\tan\phi\} \quad . \quad . \quad . \quad (24)$$

If as in § 2 the number of encounters between σ and $\sigma+d\sigma$ is $ns.2\pi\sigma d\sigma$ the total radiation is

$$ns\cdot\frac{4\pi}{3}\frac{e^2a^4}{c^3v^5}\int_0^\infty\{\phi(\sec^2\phi+2)-3\tan\phi\}\frac{d\sigma}{\sigma^4}.$$

And since by (23)

$$d\sigma/\sigma^4=\cos^2\phi\,\mathrm{cosec}^4\phi d\phi\cdot v^6/a^3,$$

the integration gives

$$ns\cdot\frac{4\pi}{3}\cdot\frac{e^2av}{c^3}\Big[\mathrm{cosec}^2\phi(1-\phi\cot\phi)\Big]_0^{\frac{\pi}{2}}$$
$$=ns\cdot\frac{8\pi}{9}\cdot\frac{e^2av}{c^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

For the radiation per cu. cm. per second we must put

$$s=\frac{\rho}{\mu\mathrm{H}} \qquad n=\frac{\rho v'}{\mu\mathrm{H}} \qquad v=\tfrac{1}{2}v' \qquad a=e^2/4m,$$

where v' is the relative velocity of the pairs of electrons. (I omit the correcting factor $1+f$). Thus

$$k\rho ac\mathrm{T}^4=\frac{\rho^2}{\mathrm{H}^2\mu^2}\frac{\pi}{9}\frac{e^4v'^2}{mc^3},$$

or since the average value of mv'^2 is $6\mathrm{RT}$

$$\mu k\frac{\mathrm{T}^3}{\rho}=\frac{2\pi}{3}\frac{\mathrm{R}b^2}{\mathrm{H}^2a\mu}.$$

Hence by (15)

$$k=\frac{2\pi}{9}\frac{\beta}{1-\beta}\frac{b^2}{\mathrm{H}\mu} \quad . \quad . \quad . \quad . \quad (26)$$

This part of the absorption-coefficient is constant throughout the star. For Capella the value is

$$k=\cdot038.$$

The contribution is thus entirely negligible.

10. Another process contributing to k is the line-absorption and emission due to excitation of an electron from its normal orbit to a higher quantum orbit.

The number of electrons thus excited at any one moment can be calculated by thermodynamics; but we have to know in addition the average time within which they emit their energy of excitation and are replaced by others which have absorbed an equal amount of energy. Not much is known as to the laws governing this emission, but

presumably, if we calculate the rate of emission by classical electrodynamics, we shall obtain an upper limit.

Kramers' treatment of the free electrons seems to have been suggested by what has been supposed to happen to these bound electrons. Its success suggests an inversion of the argument. The continuous emission predicted by classical theory being no longer possible, we may expect that it will be partitioned to correspond to the various possible orbits of inferior energy, and that each partition will only be emitted if the corresponding orbit is vacant. In particular, if the electron is in its normal orbit there is no inferior vacant orbit and no emission occurs. In any case, the classical emission should give an upper limit—which is likely to be much too high, just as in the case of capture of free electrons.

If W_1 is the negative energy of the electron in its disturbed orbit, the acceleration is $4W_1^2/Ne^2m$ and the classical emission is

$$\frac{32W_1^4}{3N^2e^2m^2c^3}.$$

For an n-quantum orbit W_1 is proportional to $1/n^2$, and since the frequency of excitation also decreases as n increases, the main contribution will come from the 2-quantum orbits.

If $W = h\nu_K$ is the negative energy of a normal K electron, its energy in a 2-quantum orbit is $W_1 = \frac{1}{4}W$, and the fraction of atoms so excited is approximately * $e^{-(W-W_1)/RT}$. The number of excited atoms per gram is thus

$$\frac{1-x}{HA}e^{-\frac{3}{4}W/RT}$$

where x is the ionisation. Equating the emission of these atoms to the absorption

$$kacT^4 = \frac{1-x}{HA}e^{-\frac{3}{4}W/RT}\frac{W^4}{24N^2e^2m^2c^3}.$$

Since $a = 48\pi\alpha R^4/c^3h^3$ ($\alpha = 1{\cdot}0823$)

$$k = \frac{1}{1152\pi\alpha}\cdot\frac{h^3}{Hm^2e^2c}\cdot\frac{1-x}{N^2A}\cdot y^4e^{-\frac{3}{4}y}$$

where $y = h\nu_K/RT$.

The function $y^4e^{-\frac{3}{4}y}$ has the maximum value 14·82 when $y = 5{\cdot}33$. Using this maximum value we obtain for the upper limit

$$k = 115000\frac{1-x}{N^2A}.$$

For elements of atomic number below 20, $1-x$ is very small because the K ionisation is practically complete in stellar conditions. For

* I neglect weights of statistical states, etc., giving factors which commonly cancel out. *E.g.* with two K electrons there is a double chance of excitation, but the unexcited electron by occupying one of the two orbits presumably blocks half of the emission.

elements above 20 the denominator is greater than 16000. Hence k is certainly less than 7; and since this is a rather extreme upper limit, k is probably of no astronomical importance.

11. If we try to make an exhaustive catalogue of the processes of energy-transfer between radiation and matter in a star, we arrive at the following list. Each process is reversible.

1. Absorption of radiation by atoms without disruption (excitation).
2. Absorption (scattering) of radiation by free electrons.
3. Absorption of radiation by atoms with disruption (ionisation), and the reverse process of combination of electrons with atoms (capture).
4. Absorption and emission at encounters of electrons with atoms without combination.
5. Absorption and emission at encounters of electrons with one another.
6. Absorption and emission at encounters of atoms with one another.
7. Ionisation of atoms by electrons, and conversely simultaneous encounter of two electrons with an atom resulting in a capture by mutual interaction.
8. Higher coincidences.

Of these 1, 3, 4, and 5 have now been discussed. No. 2 presents no difficulty; it is well known that it gives an absorption-coefficient $k=0{\cdot}2$. No. 6 can scarcely be important; the velocities of the atoms are small, encounters are infrequent, and in stellar conditions the high positive charges of the atoms prevent actual collision. No. 7 is generally supposed to be radiationless and therefore has no effect on k; in any case it can only occur comparatively seldom in a star.* It seems that these other processes cannot add anything appreciable to the absorption-coefficient (k_0+k_x) already determined.

It is perhaps worth while to gain a clear idea of why these transfers are so important for us, seeing that in thermodynamical equilibrium each transfer is almost immediately neutralised by a reverse transfer of the same kind. The reason is that in the stars the equilibrium is not quite perfect. The matter is in equilibrium, but the radiation is not; the energy of the matter is isotropic, but the energy of the radiation is preponderantly associated with outward velocity. Hence, when energy passes from radiation to matter and back to radiation it is not unchanged in quality; it is transformed from anisotropic to isotropic form. It is just this effect of hindering the outward flow of radiation by continually converting it into isotropic flow which makes the combined absorption-emission potent in determining the luminosity of the star.

* With reference to my controversy with Lindemann on this subject, Milne (*Phil. Mag.*, 47, 236) points out that even if large numbers of electrons are set free by collisions my calculation of the absorption-coefficient would be unaffected, since they would have to be captured by an equally radiationless process; but he appears to agree with Lindemann that extensive ionisation by collision actually occurs. I think this is clearly negatived by energy considerations; according to his own determination (which agrees with mine) the atoms are ionised down to a level such that it would require the average energy of 5 free electrons to remove another electron.

12. The discrepant factor $\frac{1}{8}$ between theory and observation makes it very difficult to draw a conclusion. On the one hand, considering the intricate nature of the problem, the agreement might be considered close enough to afford a general confirmation of the theory. On the other hand, the theory is clear-cut and the data are not subject to much uncertainty, so that it is difficult to see why the agreement should not have been closer. As it stands, the theory would predict a magnitude of Capella about $2\frac{1}{4}^{m}$ brighter than is actually observed. Naturally it would be much more satisfactory if more stars were available to test the theory. If we take the view that there is a real discordance the remaining $\frac{7}{8}$ of the absorption-coefficient must be attributed to some additional energy-transfer not considered by Kramers. But is it likely that some independent process would give a result so nearly of the same order of magnitude?

In this connection it seems necessary to refer to the position of my theory of nuclear capture as affected by the present investigation. Since I first suggested it, the calculation has been improved in three respects: (1) the definite acceptance of the "relativity correction"; (2) the use of astronomical results for Capella instead of uncertain statistics; (3) revision of the actual molecular weight for Capella. The effect is to improve the agreement of theory and observation, the values in § 4, $k_n = 78$; $k_a = 47$, agreeing within the limits of uncertainty.

In admitting Kramers' captures we do not definitely exclude the possibility that nuclear captures also occur. His captures occur chiefly among electrons aimed at an annular target $\sigma = 7 \cdot 10^{-10}$ to $15 \cdot 10^{-10}$ cm.* For smaller values of σ few captures occur. At about $\sigma = 3 \cdot 10^{-10}$ his theory ceases to be applicable, since the classical radiation now exceeds the initial energy; he assumes, however, that there is no recrudescence of capture in this region. At about $1 \cdot 1 . 10^{-10}$ cm. a new condition arises; the particles (which, as Kramers assumes, do not radiate) now hit the nucleus and the theory breaks down in a more radical manner. We can scarcely offer a suggestion as to how to apply the correspondence principle to these interrupted tracks. If the majority of these electrons were captured we should obtain the correct value of the absorption-coefficient.

I had attempted to test my theory by determining the emission of radiation due to nuclear capture when a stream of electrons falls on an anticathode. The result was about 100 times larger than the experimental value—an instance of equation (9). This failure is satisfactorily explained by considering that the atoms in the anticathode are unionised, so that capture of any kind is prevented; the electron has no choice but to gather up its energy and go off again. In the ionised atoms of the stars the electron can find a resting-place and the emission occurs.

But Kramers' theory reveals a serious difficulty in regard to nuclear capture. Emission by capture in a K orbit must be balanced by absorption with expulsion of K electrons; L emission must similarly balance L absorption. We know experimentally the K and L absorption, and these balance Kramers' captures; at any rate the surplus is only small.

* The numbers refer to iron at the centre of Capella; the radius of the K ring in iron is about $2 \cdot 10^{-10}$ cm.

Thus we cannot arbitrarily add a large number of nuclear captures. If electrons are captured by nuclear encounters they will have to fall mainly into M or higher orbits. This would approximately halve my calculated coefficient, since the mean energy given up on capture would be not much greater than RT instead of the mean radiation-quantum 2·7 RT assumed in my calculation; agreement of theory and observation would be further improved. But I am afraid this limitation does not tend to make the theory more credible.

Being thus driven to take refuge in higher atomic levels comparatively safe from experimental exploration, I confess that the theory of nuclear collision affords me little satisfaction, and I would rather strive to reconcile Kramers' process with the astronomical facts; yet I have some hesitation in discarding entirely a theory which agrees so well with astronomical observation in favour of one of which the agreement is as yet rather poor.

One reason against too summary rejection of the nuclear theory is that the experimental evidence seems not wholly opposed to a process of capture of electrons in high-level orbits additional to that of Kramers. The experiments of Kuhlenkampff (*Ann. d. Physik.*, **69**, 548), on which Kramers especially relies, exhibit the so-called continuous X-ray spectrum as composed of two parts A and B. Kramers deals only with spectrum A. For electrons of homogeneous velocity, spectrum B consists of a line (or fine band) emission very near to the frequency ν_0. The electrons emitting this frequency are thus left with an energy differing very little from zero, and presumably they are supernumerary electrons captured in orbits of high quantum number at or above the peripheral level. The intensity of spectrum B is far too strong to result, in accordance with Kramers' postulate, from a sweeping together of the continuous emission in the range represented by this line. If the experimental work is not misleading it would seem to indicate some extra process of high-level capture. Like any other capture process its full development can only occur in the ionised atoms of the stars.

Again, the circumstances of an electron hitting the nucleus are not dissimilar to those of a β particle issuing from a nucleus. There is some ground for believing that a β particle radiates energy during its exit,* so that it is possible that an electron penetrating into the outer parts of the nucleus and emerging again will also radiate. A relatively small loss of energy of the latter will lead to high-level capture.

An erroneous theory sometimes gives approximate results, because its formulæ are controlled by dimensional relations independent of the details of the theory. But in the present case my formulæ differ from Kramers by a natural factor $hc/e^2 = 861$—one of the few natural constants of zero dimensions. The difference is whittled down to 8 by casual factors.

Postscript.—In § 10 the elliptic two-quantum orbit (for which the classical radiation is 40 times greater) ought to have been considered. The argument is thus inconclusive.

Summary.—Applying Kramers' theory of the continuous and line emission of radiation during the encounters of electrons with atoms to the

* Andrade, *The Structure of the Atom*, 55.

star Capella, we obtain an absorption-coefficient about one-eighth of that determined from astronomical observation. So far as we can make out the disagreement is outside the limits of uncertainty, but it is difficult to be sure of this owing to the intricacy of the problem. The question whether Kramers' process is the principal process of energy-transfer in the star must be left open.

The state of ionisation in Capella is examined in considerable detail and evidence is obtained that the mean molecular weight must be near 2·2—a somewhat lower value than I have previously thought probable.

The Systematic Motions of Faint Stars. Second Paper.

By W. M. Smart.

(*Communicated by the Director of Cambridge Observatory.*)

§ 1. In a previous paper (*Monthly Notices*, 84, 3), I gave the results of an analysis of the proper motions of faint stars in 4 photographic regions in which the phenomenon of star-streaming was conspicuous. In the present paper the distribution of the proper motions in 4 additional regions is discussed; in these, too, star-streaming is strongly shown. The regions are as follows:—

Number.	Region.	R.A. (1900) h m	Dec. ° ′	No. of Stars.	Interval.
I.	ρ Persei	2 59	+38 27	65	18·0 years
II.	B.D. 30° 3639	19 31	+30 18	436	18·0 ,,
III.	Kapteyn's Special Area 12	21 57	+58 29	159	11·8 ,,
IV.	Lalande 45755	23 17	+43 33	105	18·6 ,,

In 3 of the regions the name of the central star is given. The numbers of stars whose proper motions have been measured are given in the penultimate column, and the intervals between the early plates (direct) and the recent plates (photographed through the glass) are in the last column. The interval for Region III. is rather short, but it seemed worth while to attempt to derive the proper motions in one of Kapteyn's Selected Areas (Special Plan), even although the results must be regarded as of much less weight than those of the remaining regions, and, for the present, only as provisional in character.

The method of reduction and the analysis of the proper motions are essentially those of the previous paper. The main difficulty encountered in this work is in deriving the absolute from the relative proper motions: in the absence of a sufficient number of stars in each region with well-determined meridian proper motions, one is thrown back on the mean parallactic motions of a comparatively large number of stars, with which to compare the relative proper motions. It is fortunate that one of the regions—II.—contains no less than 18 stars, for which meridian proper motions have been derived (*Greenwich Catalogue of Stars*, 1910), and it is a matter of considerable interest to compare the results of the respective procedures (see § 5 below).

Let (ξ, η) denote, as before, the co-ordinates of the origin of absolute motions (derived from the consideration of parallactic motions or of meridian proper motions) with reference to the origin of relative motions.

The following table gives the number of B.D. stars in each region which were used in deriving the values of (ξ, η) by comparing the mean relative motions with the mean parallactic motions:—

TABLE I.

Region.	No. of B.D. Stars.
I.	36
II.	75
III.	40
IV.	45

§ 2. *Distribution of the Proper Motions in Direction.*—In deriving the distribution of proper motions for each of the 4 regions, I have used the origin obtained from the consideration of parallactic motions alone. In Table II. the number of stars moving in each of the 36 sectors $\theta - 5^\circ$ to $\theta + 5^\circ$ ($\theta = 0^\circ, 10^\circ, \ldots 350^\circ$) is given. In the case of Region II. 335 stars had from 4 to 6 determinations, the remainder only 2. The column II*a*. gives the information with regard to these 335 stars, and column II*b*. with regard to all the 436 stars, the insufficiently determined stars being given half weight. The curves $r = f(\theta)$, given below have as radius vector, r, the number of stars in the 30° sectors $\theta - 15^\circ$ to $\theta + 15^\circ$ ($\theta = 0^\circ, 10^\circ, \ldots 350^\circ$). The values of r are easily obtained from Table II.

TABLE II.

Distribution of Proper Motions in Direction.

	Region.						Region.				
θ.	I.	II*a*.	II*b*.	III.	IV.	θ.	I.	II*a*.	II*b*.	III.	IV.
0°	2	6	6½	3	0	180°	1	17	20	4	5
10	1	2	2	1	1	190	4	19	21½	2	5
20	0	10	11½	1	2	200	0	13	13½	9	6
30	0	5	6½	4	1	210	2	19	22	5	3
40	1	7	9½	6	3	220	1	18	21½	6	3
50	0	11	12	12	4	230	1	7	8½	6	5
60	3	16	17½	8	6	240	2	4	6½	3	1
70	1	7	9	5	6	250	0	6	7	6	1
80	3	9	9	6	8	260	0	4	5	5	1
90	3	13	14½	2	5	270	1	3	5	5	1
100	4	16	16	4	5	280	1	3	3	3	0
110	3	16	17½	6	4	290	0	6	6½	7	0
120	4	9	11	3	6	300	1	1	1½	6	1
130	7	18	18½	3	4	310	1	4	4	0	0
140	6	8	8½	1	4	320	0	4	4	3	0
150	4	15	17	5	2	330	0	3	4	3	0
160	4	19	22	4	6	340	0	1	2	5	1
170	4	11	15½	1	4	350	0	5	6	6	1

§ 3. In the following paragraphs I have given for each region the galactic latitude, position angles of the solar ant-apex, and apices of Drift I. and Drift II., and the angular distances λ, λ_I, λ_{II} of the centre of the region from these respective apices. The positions of the apices used are:—

Solar Ant-apex	:	R.A. 90°	:	Dec. −34°
Apex of Drift I.	:	90°	:	−14°
,, Drift II.	:	288°	:	−64°.

The limiting magnitude (Harvard Scale) of the faintest stars measured has been derived from Van Rhijn's Table IV. (*Groningen Publications*, 27).

I have given also the factor F, as the units of proper motion used throughout are expressed in terms of the scale value of the measuring machine and the interval T between the pairs of comparison plates. A proper motion p, expressed in these units, is equivalent to $\left(\frac{p}{F}\right)''$ per century.

§ 4. *Region I.* (ρ Persei).—For this region we have:—

Galactic Latitude	= − 16°
Position Angle of Ant-apex	= 143°
,, ,, apex of Drift I.	= 132°
,, ,, ,, Drift II.	= 265°
λ, λ_I, λ_{II}	= 84°, 67°, 126° respectively.
Limiting Magnitude (H.S.)	= $11^m{\cdot}3$
Factor F	= 10·2.

Fig. 1 shows the distribution of proper motions of the 65 stars of this region. The curve is approximately a single drift curve in position angle 135°, practically the direction of Drift I. The feature of this diagram is the complete absence of any indication of Drift II.: this is not due to the foreshortening of the motions of Drift II. stars, for the distance of the centre of the region from the ant-apex of Drift II. is 54°. The alternative suggestion is that the velocity of Drift II. stars is systematically smaller in this region than in the sky generally. Professor Eddington has pointed out to me that in his analysis of the Boss stars, his Region III., which includes ρ Persei, provided the largest discrepancy between the observed and calculated velocities of Drift II., the former being half the latter—the respective values of hV_2 are 0.20 and 0·40 (*Monthly Notices*, 71, 37). Although the existence of some irregularity can hardly be doubted from Professor Eddington's analysis, the present result can have but a small weight in confirmation owing to the small number of stars considered. I hope that it will be possible soon to obtain further evidence from measures of proper motions of regions in the immediate neighbourhood.

In this field there are two stars with proper motions determined from meridian observations: (i) ρ Persei, for which the centennial proper motions are $\mu_\alpha = +1^s{\cdot}15$, $\mu_\delta = -10''{\cdot}8$ (Boss, *Preliminary General Catalogue*); (ii) B.D. 38°617 (Groombridge 598), for which $\mu_\alpha = -0^s{\cdot}16$, $\mu_\delta = +1''{\cdot}9$ (R. Schorr, *Eigenbewegungs-Lexikon*). The colour screen has

been used to cut down the light of ρ Persei on all plates taken directly or through the glass. The visual magnitude of B.D. 38°617 is $6^{m}\cdot8$, and consequently the large images of this bright star do not yield very reliable relative proper motions. From the absolute proper motions of these two stars, one can derive the co-ordinates (ξ, η) of the origin of absolute motions referred to, the origin of relative motions. In Table

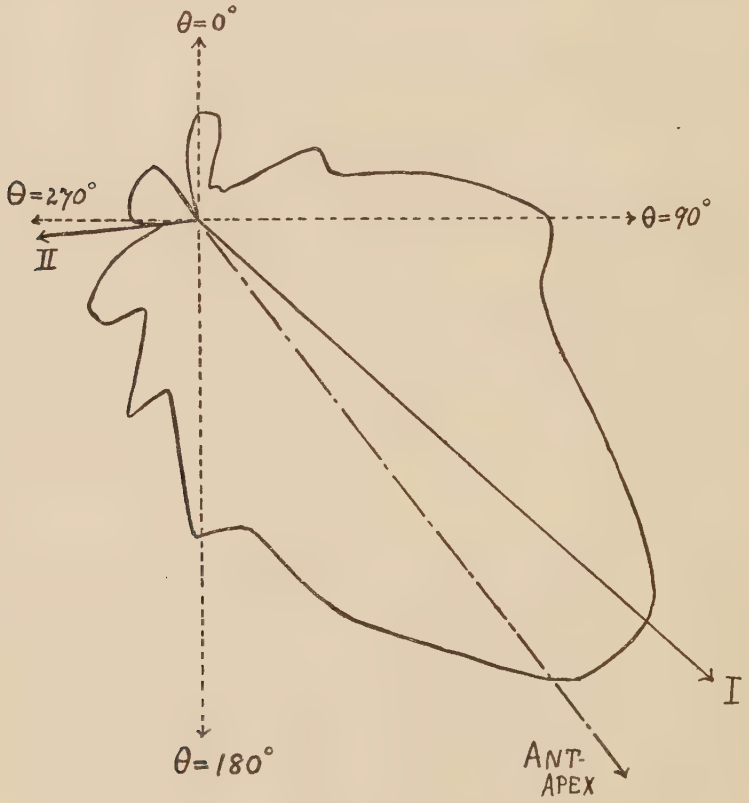


FIG. 1.—(ρ *Persei.*)

III. below, the values of ξ, η derived from the parallactic motions and from the stars (i) and (ii) are given.

TABLE III.

	ξ.	η.
From Parallactic Motions	− 8	+10
,, ρ Persei (i)	−20	+11
,, B.D. 38°617 (ii)	+13	−15

The first two lines of the Table are fairly accordant. The results for star (ii) are evidently largely discordant, and this is possibly due in

a considerable degree to the difficulty in measuring the large images of (ii) with the desired accuracy. Using the values of ξ, η, derived from the parallactic motions of 36 B.D. stars, I obtain the following values of the centennial proper motions of the two stars :—

ρ Persei	:	$\mu_\alpha = +1^s\cdot05$	:	$\mu_\delta = -10''\cdot7$
B.D. 38°617	:	$\mu_\alpha = +0^s\cdot02$	:	$\mu_\delta = -\ 0''\cdot6$.

The star B.D. 38°628 has a considerable proper motion. The values of μ_α and μ_δ deduced from the measures are :—

$$\mu_\alpha = +1^s\cdot29 \quad : \quad \mu_\delta = -1''\cdot5.$$

The magnitude of the star is 9·2 on the Harvard Scale.

Two stars whose approximate co-ordinates for 1900·0 are :—

(i) B.D. 38°629	:	R.A. $2^h\ 58^m\cdot4$	:	Dec. $+39°\ 7'$	(Mag. 8·3)
(ii)	:	$2^h\ 59^m\cdot1$	:	$+38°35'$	(Mag. 9·5)

have the following almost identical centennial proper motions :—

(i)	$\mu_\alpha = +0^s\cdot46$	:	$\mu_\delta = -2''\cdot9$	
(ii)	$\mu_\alpha = +0^s\cdot46$	:	$\mu_\delta = -3''\cdot0$.	

The position angle given by their motion is 129° as compared with 143° for the direction of the solar ant-apex.

§ 5. *Region II.* (B.D. 30°3639).—For this region we have :—

Galactic Latitude	$= +\ 4°$
Position Angle of Ant-apex	$= 108°$
,, ,, apex of Drift I.	$= 57°$
,, ,, ,, Drift II.	$= 182°$
λ, λ_I, λ_{II}	$= 160°, 153°, 94°$
Limiting Magnitude (H.S.)	$= 13^m\cdot2$
Factor F	$= 10\cdot2$.

Fig. 2 shows the distribution of proper motions of the stars in this region. For the full curve, the results from the measures of 335 stars with 4 to 6 determinations were used. There were, in addition, 101 fainter stars with only 2 determinations; these have been given half weight and the broken curve is the result of the analysis of the proper motions of the total of 436 stars. Drift II. is well marked and its direction agrees very well with that calculated. Drift I., despite the large foreshortening—the distance of the Ant-apex of Drift is 27°—is also clearly and accurately shown. There is also strong evidence of Drift O. The directions of the drifts are, on the whole, in excellent agreement with the results derived for the bright stars.

The values of (ξ, η), derived from considering the mean parallactic motion of 75 B.D. stars are $(-6, 0)$. This region lies within the limits of the *Greenwich Catalogue of Stars*, 1910, and it is possible to compare the relative proper motions derived from the photographic plates with the meridian proper motions of the Catalogue. The positions in the Catalogue were derived from Transit Circle observations between 1906 and 1914: these positions when compared with the A.G. positions (mean interval

33 years) and with the observations of Bessel and Lalande furnish the proper motions of the Catalogue. In all, there are 18 stars in Region II. with meridian proper motions given in the Greenwich Catalogue. In Table IV. below, G_x, G_y denote the components of the Greenwich proper motions expressed in scale units for the interval (18 years)

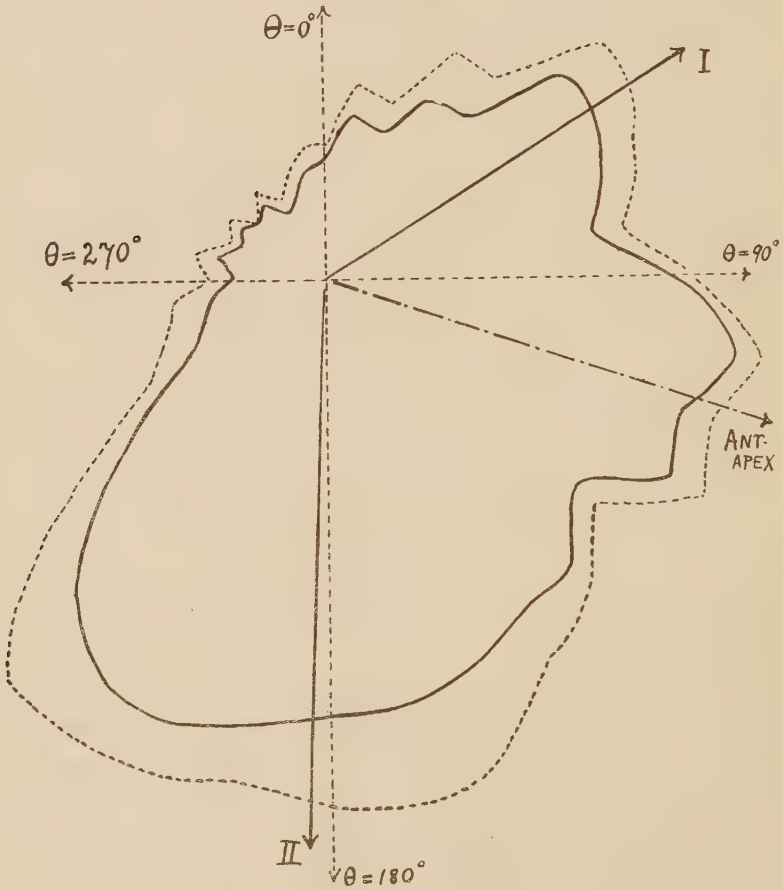


FIG. 2.—(*B.D.* 30°3639.)

between the comparison plates. M_x, M_y denote the measured components of relative proper motion in the same units. Each of the 18 stars furnishes the pair of equations

$$\xi = M_x - G_x \quad : \quad \eta = M_y - G_y$$

for determining the values of ξ and η.

The magnitudes of the third column of the table are those of the Greenwich Catalogue.

TABLE IV.

B.D.	Greenwich No.	Mag.	M_x	$G_x - M_x$	M_y	G_y	ξ.	η.
30°3611	9116	8·7	+ 9	+ 9	+ 1	−13	0	+14
3621	9132	8·8	−11	+ 1	+31	−17	−12	+48
3623	9139	8·6	− 4	+14	− 2	+ 2	−18	− 4
3625	9143	8·6	+16	−17	−10	+13	+33	−23
3629	9144	8·8	+ 7	− 4	+ 1	+ 6	+11	− 5
3632	9151	9·2	− 4	+43	−12	+ 3	−47	−15
3637	9161	9·2	−12	−11	− 7	− 4	− 1	− 3
3641	9173	9·2	−27	+ 3	−39	−67	−30	+28
3646	9191	8·2	− 7	+19	−15	+12	−26	−27
3654	9208	8·6	−18	+ 2	− 5	− 9	−20	+ 4
3658	9214	7·5	−19	+ 8	− 5	+ 9	−27	−14
3660	9227	7·9	−18	− 8	−27	− 8	−10	−19
3664	9239	9·1	+60	+47	+40	+40	+13	0
29°3628	9120	7·7	−13	+ 3	− 4	+20	−16	−24
3629	9123	9·0	+ 1	−27	− 3	−30	+28	+27
3653	9176	8·4	− 4	+16	− 7	−11	−20	+ 4
3655	9186	8·4	− 1	+41	−38	−41	−42	+ 3
3656	9187	9·1	+16	+10	− 9	−28	+ 6	+19

The values of (ξ, η) derived from the last two columns are $(-10, +1)$, and when 5 stars with discordances greater than 20 are omitted the values are $(-9, -1)$. The parallactic motions of the 75 B.D. stars on the plates give $(-6, 0)$ as the values of (ξ, η). The table shows some very large discordances. For example, the star B.D. 30°3632 gives $\xi = -47$ as compared with the mean value $\xi = -10$. This star is about 18′ from the centre of the field, and the measures of M_x show an average deviation of 4 from the mean (in terms of proper motion this corresponds to 0″·4 per century). The probable error of the proper motions of the fainter stars of the Greenwich Catalogue in Right Ascension is given as $\pm 0^s{\cdot}12$ per century, or in the units above, ± 16. I have not calculated the probable errors of ξ and η derived from Table IV.; they are evidently larger than one would wish.* In the preparation of fig. 2, I used the values of (ξ, η) derived from the parallactic motions. With the larger numerical value of ξ derived from Table IV., the figure would show greater strength in Drift I. and Drift O.; Drift II. would remain very much the same; and the directions of the Drifts would not be substantially altered.

Two stars forming a double (R.A. $19^h\ 33^m{\cdot}8$: Dec. +30° 15′) show practically the same proper motion: the values are

$$-0^s{\cdot}58 \quad : \quad -7''{\cdot}0$$

and

$$-0^s{\cdot}62 \quad : \quad -7''{\cdot}4.$$

[* The probable error of ξ calculated from the above table is only ± 15.—(*Secretaries*).]

The photographic magnitudes are almost identical, and the estimated visual magnitude of each component is roughly 11^m.

The components of the pair (Greenwich 9239, 9240: Mag. 9·1) have the following proper motions :—

$$+0^s{\cdot}50 \quad : \quad +3''{\cdot}9$$

and

$$+0^s{\cdot}60 \quad : \quad +3''{\cdot}6 \text{ respectively.}$$

The central star of the region—B.D. 30°3639—is a Wolf-Rayet star: its parallax, as determined by Russell from plates taken with the Sheepshanks telescope, is given as $-0''{\cdot}018 \pm 0''{\cdot}022$. Its proper motion, from the measures just completed, is as follows :—

$$\mu_\alpha = 0^s{\cdot}00 \quad : \quad \mu_\delta = -1''{\cdot}8.$$

The largest proper motion is that of B.D. 29°3640 (Mag. 9·7) for which

$$\mu_\alpha = +0^s{\cdot}58 \quad : \quad \mu_\delta = +14''{\cdot}6.$$

§ 6. *Region III.* (Kapteyn Special Area, No. 12).—For this region we have :—

Galactic Latitude	$= -1°$
Position Angle of Ant-apex	$= 85°$
,, ,, apex of Drift I.	$= 70°$
,, ,, ,, Drift II.	$= 201°$
λ, λ_I, λ_{II}	$= 134°, 118°, 126°$ respectively.
Limiting Magnitude (H.S.)	$= 12^m{\cdot}0$
Factor F	$= 6{\cdot}71.$

The interval between the comparison plates—11·8 years—is rather short, and the results for this region must be regarded as indicating, only very roughly, the distribution of motions of the stars. In three or four years, a further determination of the proper motions, it is hoped, will provide results of an increased order of accuracy. Nevertheless, the diagram, while showing detail in the north-west quadrant that one would hardly expect, does undoubtedly indicate the tendency of the stars to move in two distinct drifts. In fact, Drift II. is shown with surprising accuracy. Drift I. is strongly shown, but in a direction differing somewhat from that calculated.

Three stars have outstanding proper motions. The approximate positions are for 1900·0. The provisional results are :—

	R.A.	Dec.	μ_α.	μ_δ.
	h m	° ′	s	″
(i)	21 59·8	+59 7	+1·94	+14·3
(ii)	21 58·5	+58 43	−1·17	+ 2·8
(iii)	B.D. 58°2366		+1·59	+ 7·0

In the above, the correction derived from the mean parallactic motions of 40 stars has been applied. As far as I am aware, there is no star in this region whose proper motion has been determined from meridian observations.

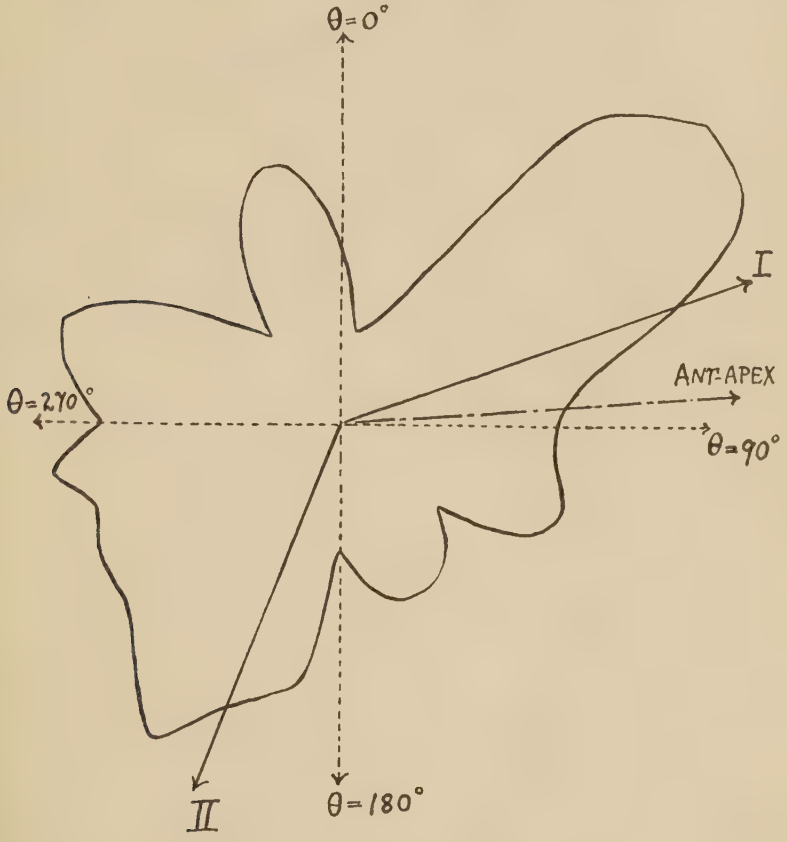


FIG. 3.—(*Kapteyn's Special Area No.* 12.)

§ 7. *Region IV.* (Lalande 45755).—For this region we have:—

Galactic Latitude	=	−16°
Position Angle of Ant-apex	=	110°
,, ,, apex of Drift I.	=	93°
,, ,, ,, Drift II.	=	206°
λ, λ_I, λ_{II}	=	121°, 107°, 118° respectively.
Limiting Magnitude (H.S.)	=	$11^m{\cdot}8$
Factor F	=	10·6.

Fig. 4 shows the distribution of proper motions in this region.

Drift II. is fairly accurately determined, but Drift I appears to deviate by about 20° from the direction calculated.

There is only one star in the region with a determined meridian proper motion—Lalande 45755—the central star. The values of μ_α and μ_δ from *Cincinnati Publications* No. 19 are

$$\mu_\alpha = +5^s\cdot81 \; : \; \mu_\delta = +22''\cdot5.$$

This star is also listed by Luyten (No. 725) in *Lick Bulletin*, 344. The

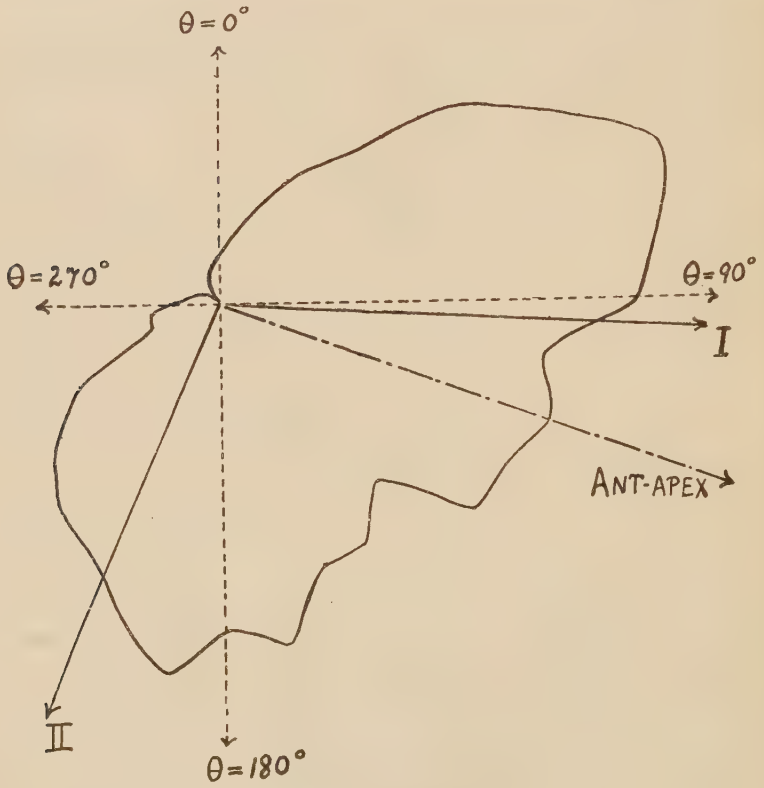


FIG. 4.—(*Lalande* 45755.)

values of (ξ, η) derived from the mean parallactic motion of 45 B.D. stars and the values derived from the above components of proper motion when compared with the measures of the relative proper motion of the star, are as follows :—

		ξ	η
Parallactic motions	:	−9	+5
Lalande 45755	:	−4	+10

The agreement is very satisfactory.

The values of μ_α, μ_δ from the Cambridge measures are $+5^s\cdot86$ and $+23''\cdot0$ respectively.

Three stars, in addition to Lalande 45755, have noteworthy proper motions. The approximate positions are for 1900.

	R.A.	Dec.	μ_α.	μ_δ.
	h m	° ′	s	″
(i)	23 16·3	+44 13	+1·23	−4·3
(ii)	23 16·0	+43 27	−0·76	−8·5
(iii)	B.D. 43°4438		+1·02	−1·9

In each case, the relative proper motions have been converted into absolute proper motions by applying the correction $+9$, -5 to the x and y components respectively.

§ 8. *Magnitudes of the Proper Motions.*—The following table gives the number of stars in the 4 regions with values of μ between various limits, μ being the total absolute proper motion along the great circle.

TABLE V.

μ.	I.	II.	III.	IV.
″ ″				
0·0 – 1·0	16	101	41	19
1·0 – 2·0	23	142	45	43
2·0 – 3·0	16	85	34	20
3·0 – 4·0	0	28	20	9
4·0 – 5·0	3	11	6	4
5·0 – 6·0	3	4	7	2
6·0 –	4	14	6	8

The numbers for Region II. are for all the stars measured, the 101 faint stars being given half weight.

Observatory, Cambridge:
1924 *January* 7.

Systematic Distribution of Solar Calcium Flocculi. II. *Life-History of Bright Solar Calcium Flocculi.* By C. P. Butler, A.R.C.Sc. (Plates 4, 5, 6.)

(*Communicated by the Director, Solar Physics Observatory, Cambridge.*)

§ 1. In a previous paper dealing with the distribution of the bright solar calcium flocculi during the years 1915–1920 (*M.N.*, 82, 334, 1922 April), it was shown that

(1) About 55 per cent. of the total number of flocculi examined exhibited some tendency to elongation.

(2) Of the flocculi showing elongation, about 80 per cent. were inclined equatorward to the west in either hemisphere, and only about 3 per cent. equatorward to the east. A considerable number of elongated formations included in the remaining 17 per cent. were roughly parallel to the solar equator, so that whether they trend to west or east has to be ascertained from a study of their development.

This form of elongation persisted during all stages of solar activity, and as it is particularly marked in the case of many small flocculi, which can be readily recognised from day to day, an examination was made of a series of such groups during the whole time of their visibility at first transit, and an example was given showing the type of variation exhibited.

§ 2. After the examination of some hundreds of flocculi it became evident that the changes exhibited could be divided into two main classes—*Gradual* and *Sudden*.

(*a*) *Gradual.*—When the flocculi may be followed throughout all the 13 or 14 days of transit, and each portion can be recognised as being substantially similar from day to day.

(*b*) *Sudden.*—When any sudden alteration takes place it may be either that some flocculus material appears in the vicinity of an existing group, or an entirely new mass is projected on a region previously devoid of visible activity. In either of these cases the manner in which the new formation rapidly alters from day to day, in many cases becoming much greater than neighbouring flocculi, is characteristic of the phenomenon.

During the passage of a complicated group it sometimes happens that minor alterations are shown which are of such a nature as to suggest gradual extension rather than sudden alteration. These minor changes may be due to several causes, such as the constantly varying presentation of formations which may have their component parts in different levels along the line of sight; or differences produced by slight variations of setting the secondary slit of the spectroheliograph on the calcium line. Although these have to be considered, it is generally evident that they are of minor importance compared with the more radical changes constituting the *sudden* phenomena.

In order to investigate the phenomena under all stages of solar activity, the inquiry was extended to embrace the eleven years, 1912–1922, including the minimum of 1913 and the maximum of 1917.

§ 3. *Method of Examination.*—The material employed for the investigation has again consisted chiefly of the excellent series of spectroheliograms in calcium (K_{232}) light taken at the Kodaikánal Observatory,

India, and kindly presented by the Director of that Observatory to the Solar Physics Observatory. As, however, in this later work the examination has extended to the whole visible hemisphere, and not only to the central meridian as in the former paper, it has been necessary to fill as many as possible of the gaps in the Kodaikánal record by reference to the Cambridge records when available, and in a few cases help has been obtained from the publications of the Tortosa Observatory.

For each day the details of every flocculus present have been carefully compared with the appearance on the preceding and following days, and classified according to the presence or absence of sudden alteration.

The features presented have been grouped in four categories:—

(*a*) *14-day Flocculi.*—Flocculi which are certainly recognisable as being due to the same disturbance for each day of their visibility, from the appearance of their preceding end at the east limb to the disappearance of their following end at the west limb (Plate 4, B; Plate 5, D). On account of their considerable extent in longitude, this time is usually somewhat longer than half of the rotation period, and may be 13, 14, or 15 days according to their size. For convenience they have been termed 14-day flocculi.

(*b*) *New Formations, Births.*—Flocculi which suddenly appear in positions where no definite indication of flocculus material previously existed distinct from the solar réseau (Plate 4, A; Plate 5, C).

(*c*) *New Formations, Growths.*—Flocculi which suddenly appear or exhibit sudden increase in extent at positions where, either from proximity to already existing flocculi, or in places where the réseau has been specially prominent, some indication of previously existing flocculus material has been suggested (Plate 5, E).

(*d*) *Deaths.*—Flocculi which suddenly disappear on the visible disc. Until this detailed examination had been made it was not known whether there might not be cases where flocculi became suddenly obliterated. None such have been observed; the only mode of extinction of flocculi is by gradual dispersal and fading.

Only one spectroheliogram is available for each day, so that the times recorded for birth, growth, etc., may be in error by any amount from 0 to 24 hours. A birth may have taken place just *after* the daily record has been secured, when it will only be shown on the photograph taken on the next day; or it may have taken place just *before* the plate for the day was exposed, in which case its recorded time will be almost correct. This is an important factor to remember when dealing with the variations shown during the early history of flocculi after formation.

Most of the phenomena accompanying the later development of flocculi which have appeared as births or growths are sufficiently similar to suggest that sudden growths are really independent births which happen to occur in close proximity to existing flocculi. In general, the growths are more difficult to deal with, as their association with other formations necessitates specially careful examination for their delimitation. For the present, therefore, births and growths are classified separately.

The observations have been summarised into the following tables showing the flocculi separated into the three categories during each month of the eleven years, 1912–22.

Bright Calcium Flocculi.

1912.	North. 14-day Flocculi.	North. Births.	North. Growths.	South. 14-day Flocculi.	South. Births.	South. Growths.
Jan.	1	1	2	2	1	1
Feb.	0	0	0	1	0	1
Mar.	1	1	0	1	2	0
Apr.	0	0	1	2	0	0
May	0	0	0	1	2	0
June	0	1	0	5	0	0
July	0	0	0	2	1	0
Aug.	0	0	0	2	0	0
Sept.	0	0	0	5	1	0
Oct.	0	0	0	5	0	1
Nov.	0	0	0	5	0	0
Dec.	0	2	0	1	0	0
Total	2	5	3	32	7	3

1914.	North. 14-day Flocculi.	North. Births.	North. Growths.	South. 14-day Flocculi.	South. Births.	South. Growths.
Jan.	4	2	0	2	1	0
Feb.	1	1	1	1	1	0
Mar.	1	1	0	2	0	1
Apr.	4	1	0	3	1	2
May	3	1	0	5	0	1
June	8	0	0	4	1	1
July	1	0	0	3	3	1
Aug.	4	1	1	6	2	2
Sept.	2	0	1	3	0	0
Oct.	5	1	3	4	5	0
Nov.	10	1	2	5	2	2
Dec.	9	2	0	8	3	0
Total	52	11	8	46	19	10

1913.	North. 14-day Flocculi.	North. Births.	North. Growths.	South. 14-day Flocculi.	South. Births.	South. Growths.
Jan.	4	0	0	1	2	0
Feb.	1	0	1	1	0	0
Mar.	1	0	0	1	1	0
Apr.	3	0	0	0	1	0
May	0	0	0	0	0	0
June	1	0	0	1	0	0
July	1	0	1	1	1	0
Aug.	0	1	0	1	0	0
Sept.	1	2	0	1	0	0
Oct.	3	3	1	1	1	0
Nov.	2	1	0	0	2	0
Dec.	1	0	1	1	3	0
Total	18	7	4	9	11	0

1915.	North. 14-day Flocculi.	North. Births.	North. Growths.	South. 14-day Flocculi.	South. Births.	South. Growths.
Jan.	10	5	0	9	1	0
Feb.	9	4	0	8	3	0
Mar.	15	0	0	10	1	0
Apr.	12	0	1	13	1	1
May	13	2	1	10	2	1
June	11	0	2	6	5	2
July	7	1	0	13	1	1
Aug.	11	3	4	15	1	2
Sept.	10	1	1	13	0	0
Oct.	13	1	3	11	0	2
Nov.	17	1	3	11	0	1
Dec.	10	1	1	8	1	2
Total	138	19	16	127	16	12

Bright Calcium Flocculi.

1916.	North. 14-day Floc-culi.	Births.	Growths.	South. 14-day Floc-culi.	Births.	Growths.
an.	15	4	3	10	1	2
eb.	15	3	2	13	1	1
lar.	13	3	2	12	2	2
pr.	12	2	0	14	1	0
May	15	0	1	15	2	2
une	12	2	1	10	1	0
uly	10	1	2	15	1	1
ug.	12	1	3	9	4	3
ept.	15	0	3	13	2	1
ct.	9	4	0	12	2	2
ov.	15	1	0	10	2	5
ec.	14	2	3	10	1	1
otal	157	23	20	143	20	20

1918.	North. 14-day Floc-culi.	Births.	Growths.	South. 14-day Floc-culi.	Births.	Growths.
Jan.	17	1	1	13	3	4
Feb.	17	1	0	20	0	0
Mar.	20	0	2	12	7	1
Apr.	13	0	1	15	0	3
May	17	1	1	13	2	2
June	19	0	2	17	1	1
July	21	0	4	16	1	8
Aug.	21	0	3	15	0	7
Sept.	11	1	4	17	1	1
Oct.	22	1	7	22	0	2
Nov.	19	1	5	19	1	2
Dec.	18	0	3	15	1	2
Total	215	6	33	194	17	33

1917.	North. 14-day Floc-culi.	Births.	Growths.	South. 14-day Floc-culi.	Births.	Growths.
an.	16	1	4	16	1	1
eb.	13	1	2	9	4	3
lar.	18	1	3	12	1	2
pr.	21	0	2	17	1	1
ay	18	1	2	15	2	0
ne	12	1	3	15	1	0
ly	14	0	4	14	1	3
g.	16	3	0	15	4	0
t.	14	1	1	13	1	2
.	16	1	2	15	0	3
v.	16	1	1	14	1	0
.	17	0	7	17	1	3
al	191	11	31	172	18	18

1919.	North. 14-day Floc-culi.	Births.	Growths.	South. 14-day Floc-culi.	Births.	Growths.
Jan.	15	0	5	11	2	3
Feb.	15	0	2	13	0	2
Mar.	16	1	2	16	0	2
Apr.	16	1	4	18	1	3
May	14	1	1	18	0	4
June	17	2	4	16	2	2
July	24	0	0	21	1	3
Aug.	16	1	2	22	1	3
Sept.	13	0	1	17	1	1
Oct.	9	1	4	20	1	0
Nov.	13	2	1	13	0	1
Dec.	18	1	3	14	0	2
Total	186	10	29	199	9	26

Bright Calcium Flocculi.

	North.			South.		
1920.	14-day Floc-culi.	Births.	Growths.	14-day Floc-culi.	Births.	Growths.
Jan.	11	1	0	9	0	3
Feb.	10	1	3	11	0	2
Mar.	15	0	2	15	2	2
Apr.	12	0	2	9	0	0
May	13	1	4	8	1	0
June	11	0	3	8	1	0
July	10	1	2	14	0	0
Aug.	9	1	1	8	1	2
Sept.	9	0	5	7	0	2
Oct.	9	0	1	5	1	2
Nov.	11	0	3	10	1	0
Dec.	11	0	2	15	0	0
Total	131	5	28	119	7	13

	North.			South.		
1921.	14-day Floc-culi.	Births.	Growths.	14-day Floc-culi.	Births.	Growths.
Jan.	10	0	0	7	0	1
Feb.	6	2	0	10	0	2
Mar.	8	3	0	12	2	0
Apr.	9	1	0	11	1	0
May	12	2	1	10	2	1
June	10	2	1	11	0	2
July	8	3	1	6	4	0
Aug.	8	1	1	7	3	0
Sept.	6	0	1	8	0	1
Oct.	7	1	1	9	0	2
Nov.	5	0	0	3	0	1
Dec.	5	4	0	4	1	0
Total	94	19	6	98	13	10

	North.			South.		
1922.	14-day Floc-culi.	Births.	Growths.	14-day Floc-culi.	Births.	Growths.
Jan.	5	0	0	8	1	3
Feb.	5	1	1	5	0	1
Mar.	12	1	0	5	2	0
Apr.	11	0	0	4	0	0
May	10	0	0	2	0	0
June	8	1	1	2	2	0
July	5	0	1	3	0	1
Aug.	3	2	0	3	0	2
Sept.	5	0	1	4	0	0
Oct.	2	1	0	1	1	0
Nov.	3	0	1	3	0	0
Dec.	5	0	2	2	0	1
Total	74	6	7	42	6	8

TABLE I.

Bright Calcium Flocculi, 1912–1922.

	North.			South.			Whole Hemisphere.		
	14-day Flocculi.	Births.	Growths.	14-day Flocculi.	Births.	Growths.	14-day Flocculi.	Births.	Growths.
	No. %	No. %	No. %	No. %	No. %	No. %	%	%	%
1912	2 (20)	5 (50)	3 (30)	32 (76)	7 (17)	3 (7)	65	23	12
1913	18 (62)	7 (24)	4 (14)	9 (45)	11 (55)	0 (0)	55	37	8
1914	52 (73)	11 (16)	8 (11)	46 (62)	19 (25)	10 (13)	67	21	12
1915	138 (80)	19 (11)	16 (9)	127 (82)	16 (10)	12 (8)	81	11	8
1916	157 (79)	23 (11)	20 (10)	143 (78)	20 (11)	20 (11)	78	11	11
1917	191 (82)	11 (5)	31 (13)	172 (82)	18 (9)	18 (9)	82	7	11
1918	215 (85)	6 (2)	33 (13)	194 (80)	17 (7)	33 (13)	82	5	13
1919	186 (83)	10 (4)	29 (13)	199 (85)	9 (4)	26 (11)	84	4	12
1920	131 (80)	5 (3)	28 (17)	119 (86)	7 (5)	13 (9)	82	4	14
1921	94 (79)	19 (16)	6 (5)	98 (81)	13 (11)	10 (8)	80	13	7
1922	74 (85)	6 (7)	7 (8)	42 (75)	6 (11)	8 (14)	81	8	11
Total	1258 (80)	122 (8)	185 (12)	1181 (80)	143 (10)	153 (10)	(80)	(9)	(11)
Yearly Average	114	11	17	107	13	14			

The outstanding feature of the tabulated data is the confirmation of the provisional statement previously made (*M.N.*, 82, 353, 1922 April), that the greater number of bright calcium flocculi were phenomena of great longevity. Except during the few months about minimum activity, the 14-day flocculi show a great preponderance over the other flocculi which are formed on the visible hemisphere.

Of almost equal prominence is the absence of any signs of flocculi becoming extinct suddenly. They are born or may develop suddenly, but when once formed in sight, they appear to live at least long enough to pass out of sight over the west limb. In not a single case during the 132 months of observation has there ever occurred an instance of a flocculus disappearing by sudden extinction of a compact mass. Every observed disappearance has taken place by gradual dispersal over a considerable area, and subsequent gradual fading of its luminosity, until it is indistinguishable from the general réseau.

A diminution of the excess of 14-day flocculi over new formations is apparent during the three years about minimum. Near the actual time of minimum we may then have the numbers of 14-day and new formations (births + growths) about equal, indicating that the disturbances are then only strong enough to survive about one rotation.

§ 4. In fig. 1 the variation in frequency of births and growths is plotted for the eleven years. Both births and growths appear to follow closely the usual variation of spot-activity for some years after minimum. After that the sun's surface becomes so active that there are few days when any parts of the flocculi zones are sufficiently free from disturbance to permit of many isolated births being recorded. Thus, during the

interval of three or four years about maximum activity, most of the new formations are described as growths. The combined curve, giving the numbers of new formations, shows only a general resemblance to the

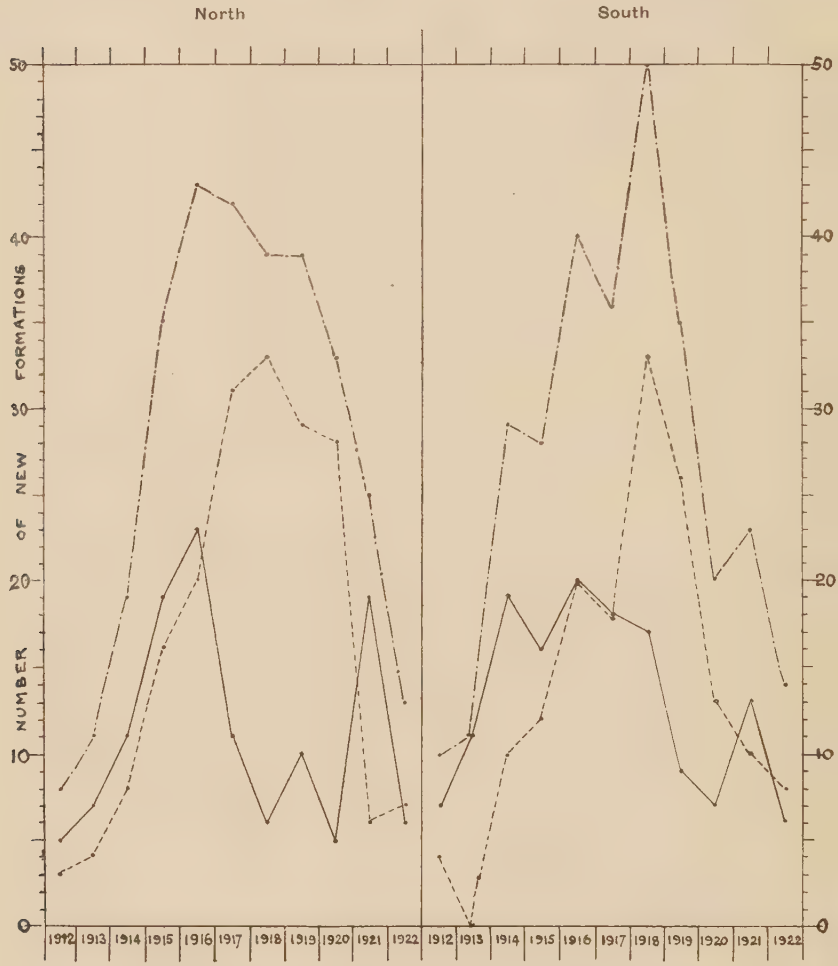


FIG. 1.—New Formations (Births and Growths), 1912–1922.

—— Births. ---- Growths. ——·—— Births + Growths.

curve of sunspot activity. The northern maximum occurs somewhat earlier than the southern maximum.

A noticeable feature of the curves showing births is the secondary maximum in 1921, indicated in both northern and southern hemispheres.

§ 5. The representative character of the summary may be seen from

a consideration of the total number of flocculi examined for the eleven years. For this purpose each flocculus was first treated as an independent unit, so that each 14-day flocculus involved a scrutiny of its features on every day of its appearance, in order to detect any material change of form. During the eleven years there were 2439 such groups, and taking only 13 days transit, corresponding to the visibility of the smaller ones, this gives a total number of 31,707 flocculi examined. Taking similarly the births and growths, and the number of days on which each was examined, we get a further 3947. In all, therefore, account has been taken of the appearance presented by 35,654 flocculi during the period 1912–1922.

§ 6. *Distribution of Outbursts of New Formations on the Visible Hemisphere.*—The accompanying Table II. shows the duration of visibility for all the new formations observed as births or growths during the eleven years 1912–1922. In no single case has a new formation been seen to be completely dispersed before reaching the western limb, consequently the table shows the frequency of outbursts in daily lunes, each of width about 14° of heliographic longitude. If we omit the easternmost and westernmost lunes, it appears that there is a fairly-marked uniformity in the frequency in the several lunes. If we take the average frequency per lune as 57, and assume that there is a similar uniformity all round the sphere, the total number of outbursts would amount to 1555. But we have seen from Table I. that 2439 groups of flocculi have been counted as being 14-day flocculi. Accordingly we must infer that many of these are "recurrent."

TABLE II.

Number of New Formations with Lives from 1 to 13 Days.

Daily Lunes.	North. Births.	North. Growths.	North. Births + Growths.	South. Births.	South. Growths.	South. Births + Growths.	Total.
East Limb 13	Entered as 14-day flocculi.						
12	3	1	4	4	4	8	12
11	6	16	22	7	18	25	47
10	15	20	35	13	15	28	63
9	10	13	23	12	14	26	49
8	8	29	37	18	13	31	68
7	22	14	36	9	18	27	63
6	11	29	40	13	19	32	72
5	13	16	29	16	13	29	58
4	14	20	34	16	12	28	62
3	6	17	23	18	16	34	57
2	9	7	16	13	8	21	37
West Limb 1	5	3	8	4	3	7	15
							603

The difference between flocculi and sunspot groups in respect of duration of life is well illustrated by even a rough comparison, which can be made with the Greenwich records of spots. In the years 1912–1919 there were 210 groups noted as recurrent, and they contributed

471 appearances on the visible hemisphere. If we regard each of these as 14-day groups—probably a large over-estimate—and add to them 132 cases of groups not described as recurrent and yet visible for 12 or more days, we have at most $471 + 132 = 603$ groups of spots which might for rough comparison be put in the category of 14-day groups.

On the other hand there were 2124 groups of non-recurrent spots, to which the 471 appearances afforded by the 210 recurrent groups should be added, making a total of 2595 items, and an exaggerated ratio of

$$\frac{\text{14-day spots}}{\text{all spots}} = \frac{603}{2595} = 0{\cdot}232 = 23 \text{ per cent. (over-estimated).}$$

Again, for the flocculi observed in the same years, 1912–1919, we have a total of 2356 flocculi, of which 1881 were classified as 14-day flocculi and 475 as new formations. We then have a ratio

$$\frac{\text{14-day flocculi}}{\text{all flocculi}} = \frac{1881}{2356} = 0{\cdot}798 = 80 \text{ per cent.}$$

The much greater proportion of 14-day flocculi shows that the flocculi in general have greater longevity than spots. The same inference is shown in a very marked way by comparing spectroheliograms taken at intervals of 27·3 days, a complete synodic rotation of the sun. The persistence of recognisable similarity of distribution of flocculi over the disc can in many cases be traced through several rotations.

As there is only one record for each day, it is impossible to discriminate as to whether any feature observed near the extreme eastern limb is a new formation or merely an appearance of an existing flocculus brought into view by the solar rotation. For this reason the first row 13 in Table II. is left vacant, and any flocculi observed there have been classified as 14-day flocculi. At the west limb it is generally certain when any feature becomes visible which was not present on the previous day.

On plotting the figures in the table their systematic nature is made more evident (see fig. 2).

§ 7. *Process of Flocculus Formation observed from Birth.*—Having seen that the feature of sudden birth or growth is consistently maintained throughout the solar cycle, we proceed to examine the details accompanying the phenomenon. The general impression from the investigation is that the subsequent development is remarkably similar in all the cases observed, the only differences noticed being apparently due to the restricted material available in only having photographs taken at intervals of 24 hours. It is thus not possible to estimate the age of a new feature nearer than one day, and it is therefore obvious that, according to whether the change has occurred 1 or 23 hours before the photograph was taken, the appearances recorded will to this extent be likely to show differences in detail. The photographs for Plates 4 and 5 were chosen to show as long series as possible, though the flocculi seen as births have already reached the elongation stage.

Bearing this in mind, the sequence of observed phenomena is as follows in most of the cases observed:

(1) *Network of Solar Réseau without Outstanding Feature.*—On a certain day we observe some region of the solar surface to be without any signs of massed flocculus beyond the usual mottling or fine-grained solar réseau (Plate 4, December 17; Plate 5, October 14).

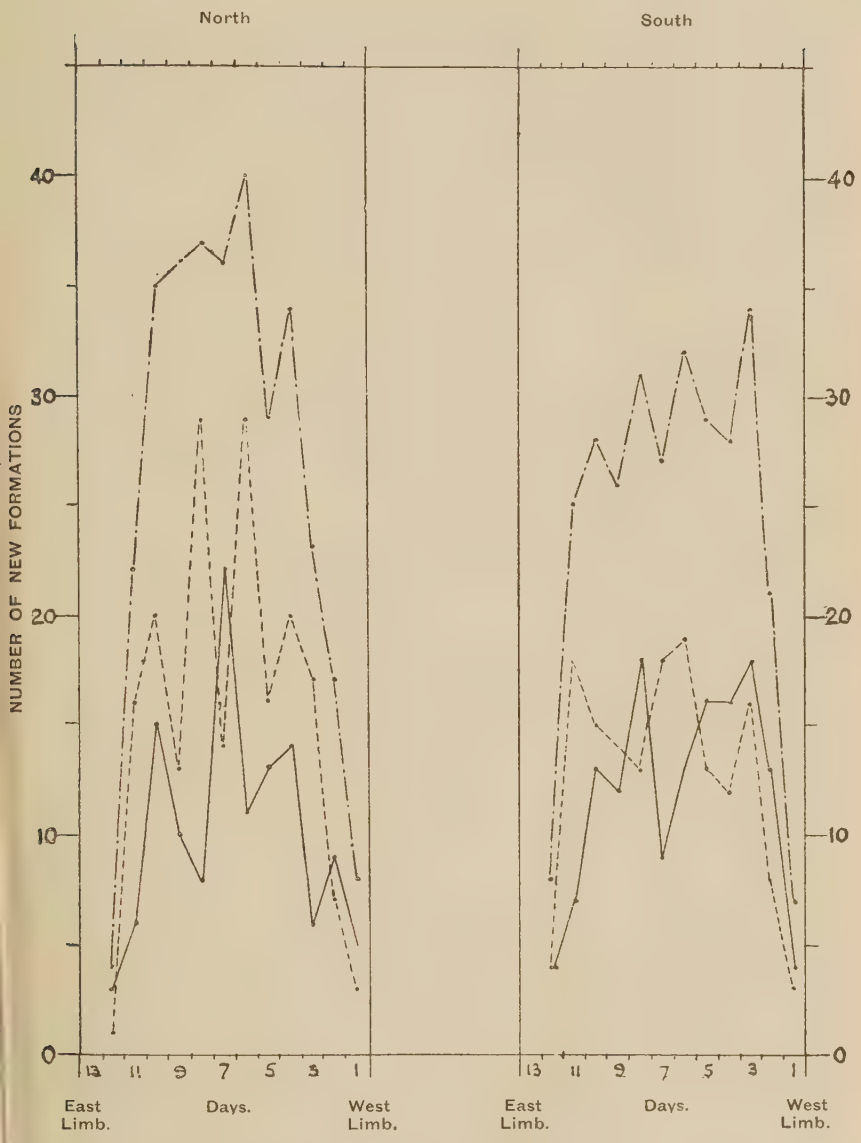


FIG. 2.—Life of New Formations (Births and Growths), 1912–1922.

—— Births. - - - - Growths. ——·—— Births + Growths.

(2) *Sudden Appearance of Bright Mass.*—On the next day's photograph we find a new bright floccular mass present, and if caught early enough, it is usually small, almost circular, and compact. In the majority of cases it is unaccompanied by any signs of spots. If the new formation occurs remote from any similar disturbance, it is called a birth (Plate 4, December 18, A; Plate 5, October 15, C); if it is in the neighbourhood of an existing flocculus, it is termed a growth (Plate 5, October 18, E).

(3) *Elongation at Various Inclinations.*—On the next day there are in most cases signs of elongation with inclination to the parallel of latitude, but in only a few cases are there any signs of spots.

(4) *Formation of Spot-Centres.*—On the third day of visibility there is generally well-marked elongation, more or less inclination to the parallels of latitude, and frequently spots are visible, it may be in the preceding end only, or at each end of the flocculus.

§ 8. So far all the flocculi observed from birth appear to follow the same sequence. The general tendency is for nearly all newly-formed flocculi to increase in area during their first passage across the disc to the west limb. For the first few days there may be differences in degree of the changes shown, some flocculi undergoing very rapid increase in area, others only slightly increasing.

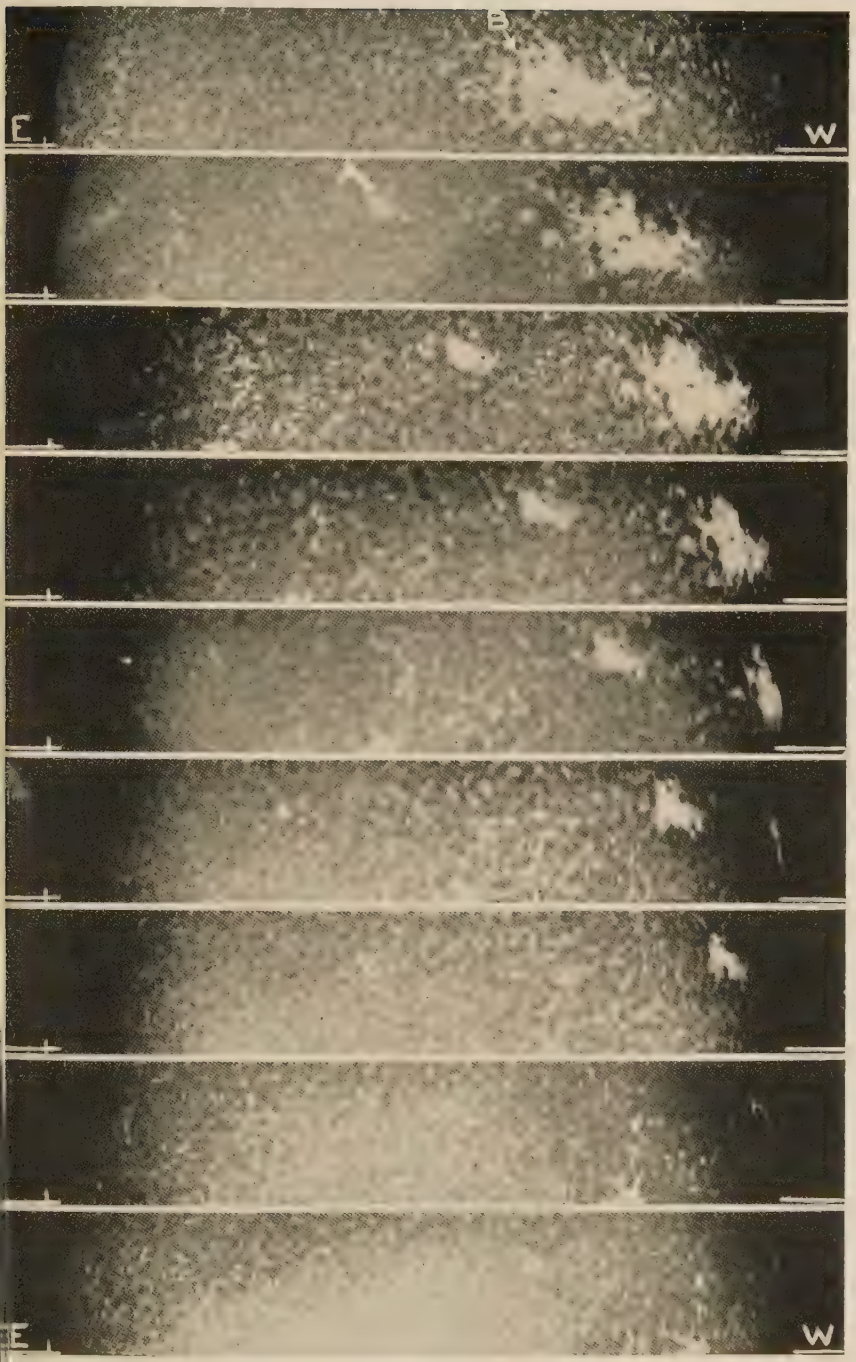
The inclination developed soon after birth is found to be in close accordance with that shown by the 14-day flocculi and described in the earlier paper. Over 42 per cent. of the inclinations observed are systematically equatorward to the west in either hemisphere, with a very small number, 2 per cent., of negative inclinations in the opposite sense, equatorward to the east. Of the 55 per cent. classed as doubtful, many are elongated, but nearly parallel to the direction of rotation; and a large proportion are included in those formed on the western half of the disc, so that their true structure is not so easily determined by reason of increasing foreshortening.

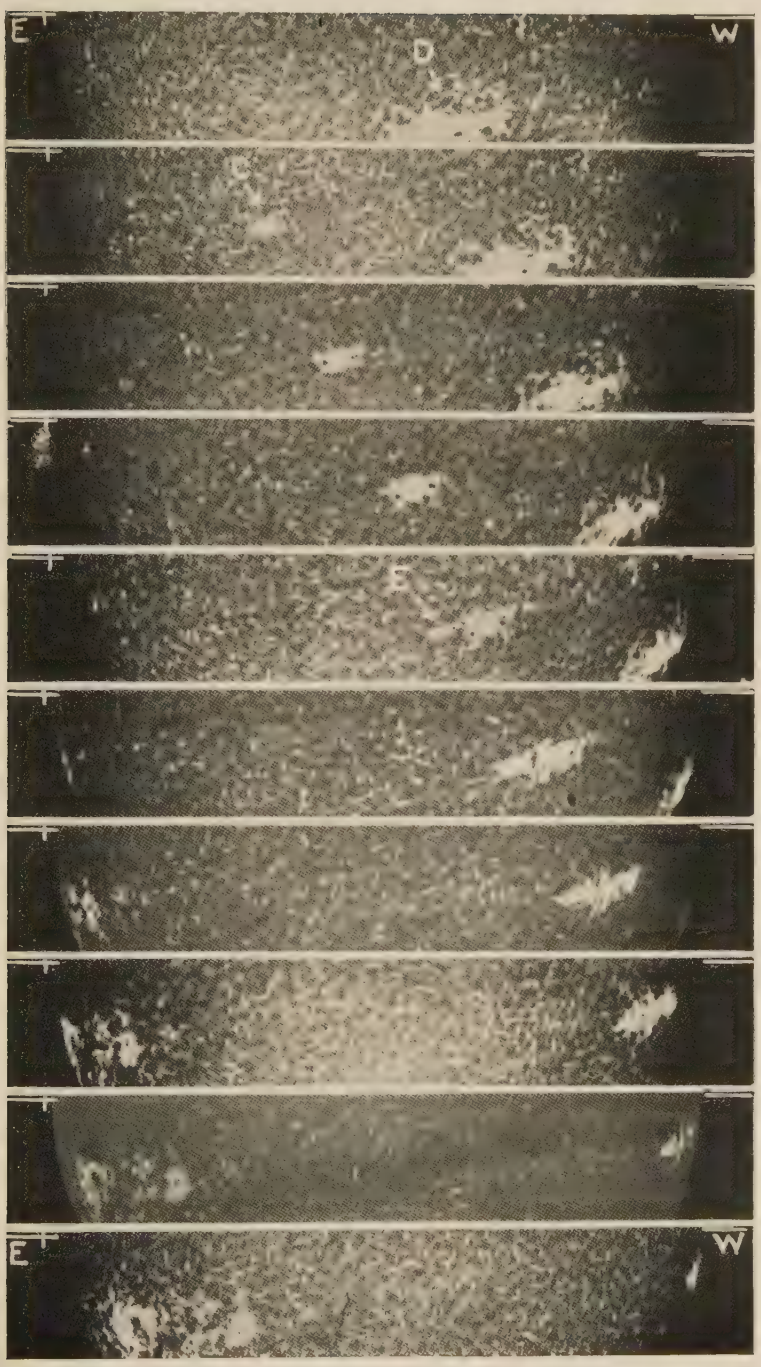
The following Table III. gives the details for each of the years under review.

TABLE III.

Showing Number of Inclined Groups in New Formations.

Year.	New Formations.	Inclination Equatorward to West.	Doubtful, Including Parallel Groups.	Inclination Equatorward to East.
1912	18	5	12	1
1913	22	3	19	0
1914	48	17	31	0
1915	63	29	34	0
1916	83	38	44	1
1917	78	37	38	3
1918	89	38	48	3
1919	74	31	41	2
1920	53	21	31	1
1921	48	22	24	2
1922	27	15	12	0
1912–1922	603	256	334	13
		42·5 per cent.	55·4 per cent.	2·1 per cent.


North
E
W
17 Dec. 1
18
19
20
21
22
23
24
25 Dec. 1
E
W



South

14 Oct. 1920

15

16

17

18

19

20

21

22

23 Oct. 1920

§ 9. *Relationship of Flocculi and Spots.*—The evidence afforded by this examination of all the new formations recorded during eleven years appears to be decisive regarding the order of formation of spots and flocculi. In no single instance has anything of the nature of a spot been observed without some trace of accompanying flocculus. There are, however, many flocculi which exist for long periods without any sign of spots being visible. The majority of cases appear to consist of the formation of a small bright flocculus area, irregular or nearly circular at first, but shortly afterwards beginning to elongate, and then only does there follow the development of spot centres.

There is thus abundance of evidence to indicate that the solar flocculi may be independent of the spots, and that their life-history is considerably different. It has also been shown that the calcium flocculi are formations of relatively great stability, while the spots appear to be secondary phenomena produced by subsequent action in the flocculi masses.

While the systematic elongation of the bright calcium flocculi indicates some persistent *stream* action, the spectroheliographic structure over spot groups usually gives some indication of disturbances involving *vortex* motion.

§ 10. *Solar Activity Indications from Sunspots and Calcium Flocculi.*—Most of the work on the variation of solar activity has hitherto been based on the changes in spot numbers or areas, and to some extent on prominence observations. Near the time of minimum the number of spotless days becomes so great that there has usually been some doubt as to the sharpness of estimation of the minimum epoch.

An attempt to employ the flocculi areas for determining the solar activity was made at the Solar Physics Observatory in 1907, soon after the commencement of the receipt of regular series of spectroheliograms from Kodaikánal, and a preliminary report made on the results for the three years 1906–7–8 was published by the Solar Physics Committee in 1913. This showed that, whereas the activity exhibited by flocculi and sunspot areas was very similar, it was possible to follow the flocculi on many occasions when spots were not present.

Even during the time about maximum there are many occasions when evidence of activity is shown by the presence of flocculi, although no spots may be present on that particular region of the solar surface (*M.N.*, 82, 339, Plate 8, 1918 February 11).

A short note to this effect was included in the report made to the Solar Union meeting at Bonn, 1913 (*Trans. Internat. Union Solar Research*, 4, 110, 1914), in which it was stated that it seems quite certain that the variations of flocculi areas will give more information than the present criterion of sunspot areas, as the objects to be measured are, in general, of much greater extent; and, moreover, *they are present long after the spots have disappeared during the minimum epoch.*

The following Table IV. regarding the activity near minimum shows a comparison of the numbers of days on which there are no spots and no flocculi respectively. For instance, in 1914, while there were 143 spotless days according to the Greenwich Publications, there were only seven days without flocculi.

TABLE IV.

Number of Days Recording

	1912.		1913.		1914.	
	No Spots.	No Flocculi	No Spots.	No Flocculi.	No Spots.	No Flocculi.
Jan.	31	14	20	2	24	0
Feb.	29	9	19	5	20	7
Mar.	19	9	29	3	22	0
Apr.	20	13	27	5	1	0
May	13	6	31	29	18	0
June	14	0	30	12	15	0
July	17	0	25	7	8	0
Aug.	30	10	31	14	15	0
Sept.	11	2	27	3	8	0
Oct.	21	0	21	0	10	0
Nov.	26	0	28	0	2	0
Dec.	17	9	23	7	0	0
Year	248	72	311	87	143	7

The accompanying diagram, fig. 3, shows this difference in the available records, the dip in each of the curves representing absence of spot and flocculus activity respectively. There is only a single indication of minimum of flocculus activity, whereas the spot curve exhibits four or more minima.

Reference to the yearly summary for 1913 (p. 136) will show how sharply the minimum epoch is defined during *May* of that year. On 29 days of that month no activity is recorded, either of established 14-day flocculi, births, or growths. It must be explained that in making the monthly summaries the tabular entries for 14-day flocculi are based on the day of central meridian passage. On the first two days of May 1913 there were the remnants of a group whose meridian passage had been recorded during April, so that it does not appear in the table under May.

§ 11. *Dispersal of the Flocculi Masses.*—It has been explained (§ 3) that the "*Deaths*" group was retained on the possibility of occurrence of *sudden* disappearances of flocculi, corresponding to the sudden appearances of births and growths. So far as this investigation goes, the evidence is that flocculi do not die in that manner, it having been noted that no satisfactory evidence of any sudden disappearance has been found during the whole of the eleven years of observation under discussion. This differentiates the phenomenon from that of the death of spots, where we have numerous cases of sudden disappearance as well as sudden formation.

But we know that flocculi do disappear. The possibility of any different effect on the side of the sun away from the earth is rendered improbable by the observations on recurrences. We have then to inquire what becomes of these great masses of bright material.

As we are debarred from following the progress of the groups for about 14 days after their disappearance round the west limb, special care is necessary in attempting to recognise them about the time they are expected to reappear at the east limb, on the assumption that they may have survived up to that time. In the case of flocculi this is perhaps more difficult than in the case of spots, owing to their greater variation in shape and size.

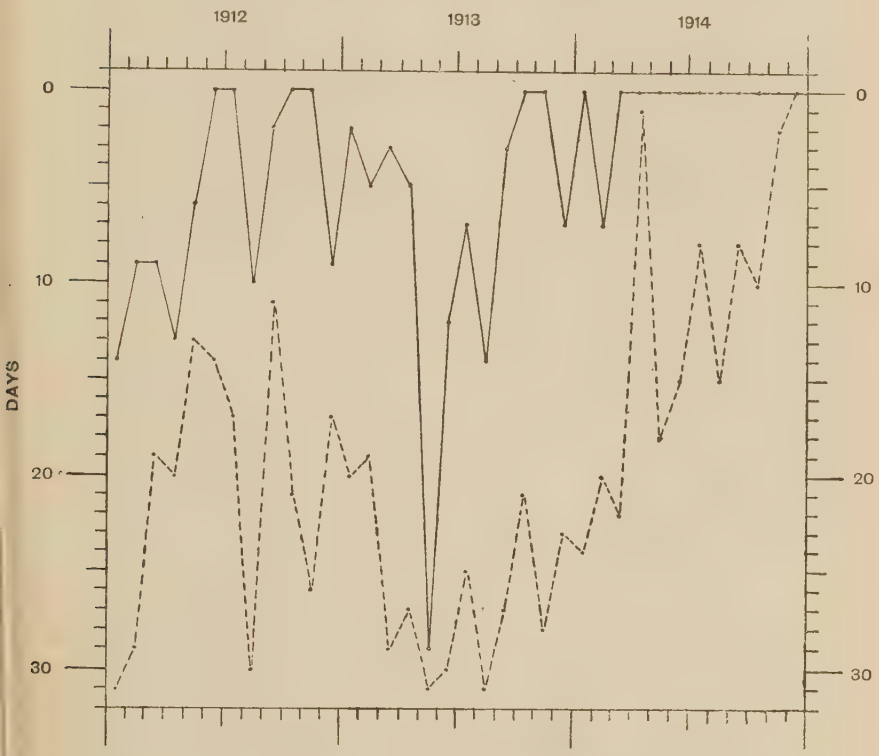


FIG. 3. —— Number of Days in each Month with no Bright Calcium Flocculi.
---- ,, ,, ,, no Spots.

By first confining attention to cases in which there appears to be no doubt of their being true recurrences, it is possible to obtain reliable evidence of the mode of development of many flocculi which live for long periods. In the selection of a series of photographs to illustrate the history of flocculi groups at successive recurrences, account has been taken of two factors: (*a*) recurrence of established flocculi, and (*b*) new formations, which to some extent are effective in opposite directions.

When the general activity is small and isolated disturbances occur, they may not be of long enough duration to give the necessary comparative measurements.

When the activity is great, long series of recurrences are easy to find, but they are frequently complicated by interference from new disturbances in their immediate vicinity.

For suitable plates, therefore, a selection has been made of a medium length of recurrence, in which perhaps the differences are not quite so pronounced as in longer series, but they show more clearly the type of effect noticed in all.

If we then examine any flocculus on its recurrence after successive rotations, a remarkable systematic order of change is observed. At each recurrence the material constituting the flocculus is seen to be more and more dispersed, without any apparent diminution of the total occupied area visible, while the general characteristics as regards inclination and position may still be recognisable throughout the recurrences (Plate 6). Ultimately, however, the process of gradual dispersal and fading renders the mass so little differentiated from the usual mottled solar surface that the flocculus is taken to have disappeared. This is the only form of extinction of flocculi masses which has been observed during the eleven years.

The study in this manner of recurrent groups according to their varied degree of dispersion or scattering at each apparition affords convincing evidence of the reality of association of many of the extended masses, which, from an isolated observation, might not appear to be due to a single original disturbance.

In the former paper (*M.N.*, 82, 351) many of the larger flocculi formations were divided and treated as separate groups in the determination of their axes to the parallel of solar latitude. From the results now given of the extent of many of these features it will be possible considerably to reduce the scatter of inclination axes by taking the mean inclinations of the groups.

§ 12. *Summary.*

(1) The number of new formations (sudden births and growths) of bright calcium flocculi on the visible hemisphere is small compared with the number of flocculi observed in complete transit over the disc. For the years 1912–1922 the number of new formations was 603, while 2439 flocculi were observed in complete transit from limb to limb.

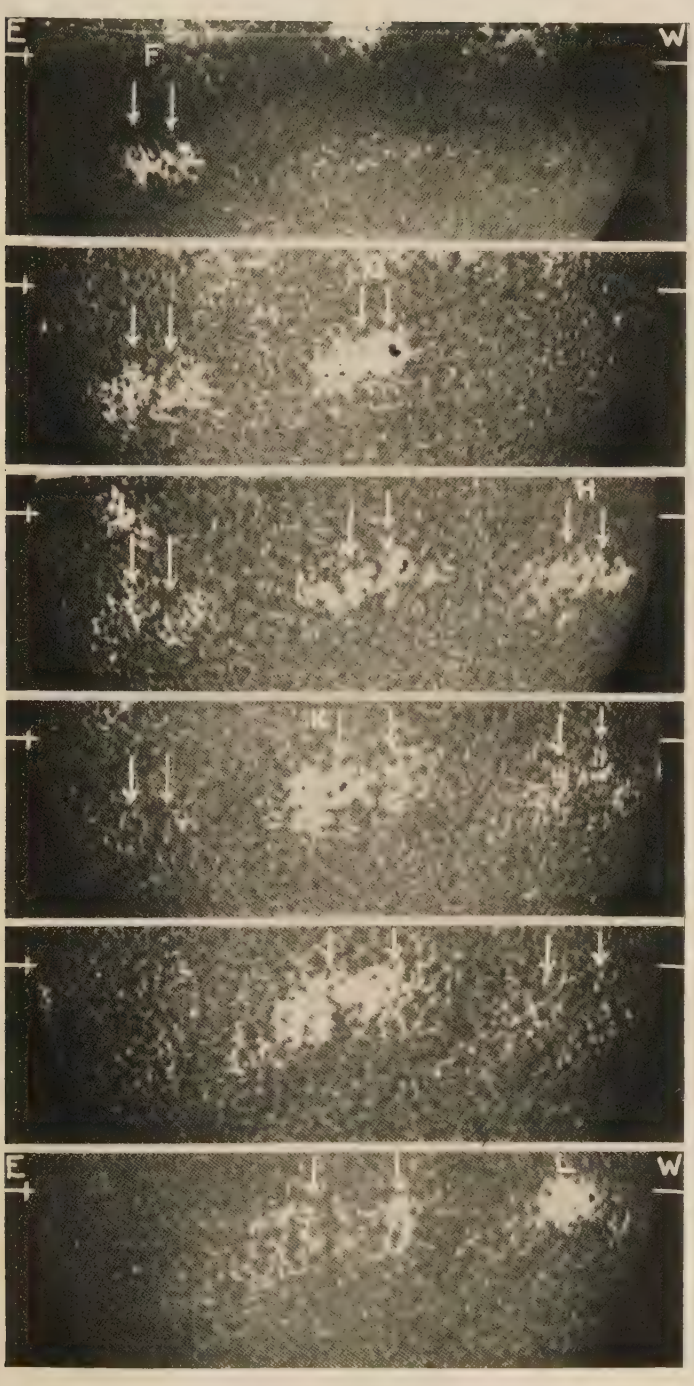
(2) The sequence of appearances in the early history of flocculi appears to be chiefly of one type, consisting of the following stages:—

(*a*) Sudden appearance of small round flocculus where no sign of activity previously existed.

(*b*) Elongation, usually with inclination equatorward to west, but sometimes parallel to equator, or in a few cases equatorward to east.

(*c*) Development of spot centres in preceding, median, or following part of flocculus.

(3) All the flocculi born on the earthward hemisphere live long enough to pass over the west limb, and, in the majority of cases, with area greater than that at birth. Only a few sudden disappearances of



South

10 Oct. 1920

6 Nov. 1920

3 Dec. 1920

30 Dec. 1920

26 Jan. 1921

22 Feb. 1921

flocculi have been suspected, and these in either very small masses or complicated portions of irregular groups.

(4) Numerous instances occur of flocculi being present without any accompanying spots, but no instances of spots occurring without some accompanying flocculus.

(5) Throughout the whole of the examination of recurrent flocculi it is found that all show a tendency for the flocculi masses to become more and more dispersed at each successive recurrence, until finally the area becomes indistinguishable from the solar réseau.

This is the only form of disappearance observed for bright flocculi. There are no sudden deaths.

I have to express my thanks to the Director, Professor Newall, for advice during the investigation, and to Mr. Manning for help in preparing the illustrations.

Solar Physics Observatory,
Cambridge.
1924 *January* 11.

Description of Plates.

Plate 4.—*Birth and Subsequent Development of Northern Flocculus*, 1921 *Dec.* 18.

In a region vacant on Dec. 17, there is on Dec. 18 a prominent flocculus
A. This becomes elongated and inclined \
B is a typical 14-day flocculus during the last six days of transit.

Plate 5.—*Birth and Subsequent Development of Southern Flocculus*, 1920 *Oct.* 15.

In a region vacant on Oct. 14, there is on Oct. 15 a prominent flocculus
C. This becomes elongated and inclined /
D is a typical 14-day flocculus during the last seven days of transit.
On Oct. 18 there is a sudden growth
E in the wake of the flocculus born Oct. 15. This growth becomes elongated and inclined /.

Plate 6.—*Increased Scattering of Flocculus Material at Successive Recurrences.*

	South-East.	Near Centre of Disc.	South-West.
1920 Oct. 10	Double flocculus **F**, fairly compact.		
1920 Nov. 6	partly scattered	New flocculus **G**, fairly compact.	
1920 Dec. 3	more scattered	breaking	New flocculus **H**, partly broken.
1920 Dec. 30	very scattered	partly broken + possibly new portion **K**	scattering
1921 Jan. 26	only just recognisable	scattering	very scattered
1921 Feb. 22	merged in réseau	very scattered	very scattered + new floculus **L**

Determination of the Deflection of Light by the Sun's Gravitational Field from Observations made at Cordillo Downs, South Australia, during the Total Eclipse of 1922 September 21. By G. F. Dodwell, Government Astronomer, S. Australia, and C. R. Davidson, of the Royal Observatory, Greenwich.

I. *The Expedition.* By G. F. Dodwell.

1. *Preparations.*—The solar eclipse of 1922 September 21 was total along a track passing from the east coast of Africa through the Maldive Islands and Christmas Island, striking the north-west coast of Australia at Wallal, passing thence through the centre of Australia and the extreme north-east corner of South Australia to southern Queensland, and emerging from Australia near Lismore, on the north coast of New South Wales.

Preparations for observing the eclipse in the interior of Australia were not commenced till 1921 October, as it was not known till then that transport and other difficulties could be overcome. The managers of Cordillo Downs sheep station in the far north-east corner of South Australia, near the Queensland border, in longitude $9^h\ 22^m\ 32^s{\cdot}0$ E., and latitude 26° 43′ S., extended a cordial welcome, through Professor Kerr Grant, for an expedition to observe the eclipse there.

An Eclipse Committee, with Sir George Murray as president, was formed at Adelaide in 1921 November.

The Beltana Pastoral Company, owners of the Cordillo Downs Station, through the Chief Director, Mr. Peter Waite, generously offered to provide transport from Farina, the nearest point on the railway, to Cordillo Downs, a distance of 400 miles, and return, with hospitality and assistance to members of the expedition at the homestead. Instruments had to be taken by camel, but observers were able to go across country by motor car.

The English Expedition was included in the invitation, but to our regret it could not be accepted, as their preparations for going to Christmas Island had been completed.

The principal object of the South Australian Expedition was to photograph the stars near the eclipsed Sun, in order to test the Einstein prediction of the deflection of light in its passage through the Sun's gravitational field; secondly, to photograph the corona, and, if possible, to obtain some spectroscopic observations; also, a programme of magnetic work was included, in conformity with Dr. Bauer's suggestions.

As there were no instruments at Adelaide suitable for the purpose of taking the Einstein photographs, applications were made abroad for the loan of instruments for obtaining these and other observations.

As a result, through the kindness of Dr. Heber D. Curtis, Director of Allegheny Observatory, U.S.A., we were afforded the use of the Allegheny quadruplet camera. The polar axis mounting, driving clock, and driving arm for this instrument were kindly lent by Dr. W. W. Campbell, Director of the Lick Observatory, who also sent us one of the Lick long-focus coronagraph lenses (6 inches aperture and 40 feet focal

length), together with the fittings for mounting it according to the Schaeberle method. Dr. Campbell also sent a short-focus lens of $2\frac{1}{8}$ inches aperture and 60 inches focal length for long-exposure photographs of the corona. Both Dr. Campbell and Dr. Curtis took a great interest in the expedition and greatly assisted us by their advice in all details. I would like to mention that Dr. Campbell, who wished that photographs of the corona might be taken with the same type of instrument at places widely distant from one another, in order to obtain evidence, if any, of rapid change of coronal structure, and had in view, amongst other things, the lower altitude of the Sun at Cordillo Downs than at Wallal, where his expedition was to be located, gave to our expedition the larger and better of the two 40-feet coronagraph lenses belonging to the Lick Observatory.

Dr. L. A. Bauer, Director of Department of Terrestrial Magnetism of the Carnegie Institution of Washington, also offered to the Cordillo Downs Expedition a grant of 100 dollars to facilitate the Magnetic Observations, which were included in the eclipse programme and were carried out by Mr. A. L. Kennedy, Chief Assistant at Adelaide Observatory.

A good series of photographs of the corona, with varying exposures, was obtained with the Lick coronagraph, and an account of these and other results will be given in the general report of the expedition.

At the Rome Congress in 1922 May, I discussed our eclipse programme with Dr. O. J. Lee, of Yerkes Observatory, Dr. H. D. Curtis, and others, and later on, at Greenwich, the Astronomer Royal and Mr. Davidson gave me invaluable advice resulting from the experience gained from the historic eclipse photographs taken at Sobral and Principe in 1919 May, which first showed the existence of the Einstein deflection. In accordance with their advice it was decided to endeavour to take four photographs with the Allegheny quadruplet, with exposures depending on results to be obtained prior to the eclipse; and on two of the plates a check field, about 5° distant from the Sun, was to be photographed, if possible, during the eclipse. A cablegram was sent to Mr. Kennedy, who was *en route* to Cordillo Downs with the instruments, asking him to take preliminary photographs about half an hour after sunset, as soon as possible after his arrival at the station.

On my way back to Australia, through America, I received a copy of Mr. Davidson's paper, giving details of suitable check fields adjacent to the eclipse field (vide *Observatory*, 1922 July, p. 224). I also received further helpful suggestions from Dr. Curtis, during a visit to the Allegheny Observatory, and from the astronomers at the Sproul, Washington, Mount Wilson, and Lick Observatories.

2. *The Allegheny Quadruplet Camera.*—This instrument is described by Schlesinger and Hudson in *Publications of the Allegheny Observatory*, 3, No. 9. A few details taken from this description may be given here. It was first used for a determination of star positions in the region of Coma Berenices. The probable error of measurement of a single image on these plates was found to be

in Right Ascension $\pm 0''.171$,
in Declination $\pm 0''.176$.

Afterwards, Dr. Schlesinger used it at Denver for making a Zone Catalogue by the photographic method, with excellent results.

The objective, made by the Brashear Company, consists of four lenses, forming two achromatic components, which are duplicates of each other and are separated by about 11·8 inches. The two lenses in each component have the same curvature on their inner faces, and are separated only by the thickness of a postage stamp.

The diameter of each lens is 4·1 inches, but a diaphragm between the two components and close to the optical centre of the whole combination reduces the effective aperture to 3 inches. This arrangement ensures a field of full illumination of more than 10° in diameter. The focal length of the quadruplet is 64·2 inches. This is a little less than half that of the telescopes that are being used for the Astrographic Catalogue, and corresponds to a scale of 0·00031 inches (0·0079 mm.) to 1 second of arc. The ratio of focal length to effective aperture is a little more than 21. Dr. Schlesinger considers this favourable to the performance of the objective over a large field. Images within 2°·5 of the axis are small and sensibly round, the diameter of the smallest image fully exposed being 0·03 mm. Beyond this distance the images begin to elongate in a radial direction, but they have the merit of remaining symmetrical, their densest portion or nucleus being always in the centre.

By means of a special contrivance of springs and adjusting screws, the lenses are fixed with great rigidity in the cell. The adjusting screws allow them to be collimated very accurately with respect to each other.

The brass lens-holder is screwed into a solid plate of steel about 14 inches square; this is connected with the equally rigid plate end by a skeleton tube of steel lattice work with lengths of strong angle iron at the four edges. The tube is covered with dark cloth, rubberised on the inner surface. Extra black cloth coverings were used at Cordillo Downs to ensure full protection from any stray light.

The plate-carrier is attached to the framework of the camera by means of four clamping and adjusting screws, which enable the plate to be tilted in any direction, so that it may be made exactly perpendicular to the optical axis of the lenses. An auxiliary apparatus is provided to secure this adjustment or to measure the exact position of the "base point" on the plate. The plates are 7 inches by 8 inches, giving a field of nearly 7° by 8°.

For the Cordillo Downs eclipse photographs the plates used were sent out from the Eastman Kodak Company, and were of plate-glass $\frac{1}{8}$th inch thick, coated with double coating of Seed 30 emulsion. They were not backed, the double coating being a sufficient protection against halation. Two brass plate-holders were supplied from Allegheny Observatory, and two others of wood and metal were made at Cordillo Downs to save time in changing plates. The camera was mounted so that the longer side of the plates corresponded with declination. For the eclipse photographs the form of mounting suggested by Dr. Campbell and Dr. Curtis, and used by them with great effectiveness in various eclipse expeditions, was adopted. The camera tube was rigidly fixed to a very strong wooden framework forming the polar axis. The camera

was screwed and bolted to this polar axis at an angle such that it would point to the declination of the eclipsed Sun. Movement was thus restricted to right ascension, and this had the advantage of increasing the rigidity of the instrument. Roller bearings, supplied from Lick Observatory, connected the polar axis with the north and south supporting piers. These were constructed of solid timber, firmly bolted and braced, and sunk into concrete foundations. An iron collar, with R.A. clamping screw, was attached to the north end of the polar axis; to this a driving arm, 10 feet 2 inches in length, was screwed. The distant end of the driving arm was provided with two pulley wheels travelling on an inclined metal plate or track, screwed to a strong support mounted in concrete. An excellent driving clock, supplied from the Lick Observatory, regulated the movement of the driving arm, which had a length of travel corresponding to about 1 hour of movement in right ascension. A slow-motion device, controlled from the guiding telescope end by means of an endless cord, was also provided.

Two finding or guiding telescopes were attached to the camera, the longer one, with micrometer eyepiece, pointing 1° 11′ north and 1° west of the optical axis of the camera, so that the guiding star β Virginis would be central in it when the camera was pointing to the eclipsed sun; the shorter one pointing at such an angle as to bring Spica into the centre of its field of view when β Virginis was in the correct position in the main guiding telescope. This ensured exact and rapid setting of the camera just before totality, or as soon as the bright star Spica became visible in the small finder.

3. *Expedition to Cordillo Downs.*—Mr. A. L. Kennedy, who made all the preparations and assembled the instruments at Adelaide during my absence at the Rome Conference, left for Cordillo Downs with the equipment on 1922 May 31. Unfortunately the camel waggons which were to have taken the instruments from Lyndhurst Siding to Cordillo Downs were not available, and the journey had to be undertaken by pack camels. This gave Mr. Kennedy a very arduous task, as the camels had to be loaded and unloaded every day. Cordillo Downs was reached on July 20. During the latter part of the journey and in the preparations at Cordillo Downs Mr. Kennedy was assisted by Mr. A. G. Appleby. The manager of the Cordillo Downs Station (Mr. C. F. Murray), kindly supplied extra assistance in erecting the instruments, and placed at the disposal of the expedition the resources of this exceptionally well-equipped sheep station. By August 29, when Mr. E. Thrum and I arrived, the work was very well forward; the Allegheny quadruplet had been erected and was in good adjustment, and several preliminary photographs of the Eclipse field had been taken. These were a good guide as to the exposures required during the eclipse, and some of them have been used as comparison plates for measuring the Einstein deflection. The 40-feet coronagraph, wireless-receiving installation, and other instruments had also been erected: and indeed it must be said that the success of the expedition was largely due to Mr. Kennedy's excellent work and thorough supervision of all the details. Professor Grant, Joint Secretary of the Eclipse Committee, reached Cordillo Downs on September 6, and shared with me the work with the Einstein instru-

ment. Later on, at Adelaide, he gave his valued co-operation in the development and preliminary examination of the plates. Mr. A. L. Nairn also kindly gave his assistance in the preliminary examination. On the day of the eclipse we were assisted in taking the Einstein photographs by Mr. and Mrs. Murray and by Mr. K. Dixon, of Professor Woolnough's exploring party, which arrived about a week before the eclipse. Mr. T. E. Barr Smith, Chief Director of the Beltana Pastoral Company, since the regretted death of Mr. Waite, reached Cordillo Downs with Messrs. Ive (Managing Director) and Adamson (Secretary of the Company), and called the time during the eclipse. In this he was assisted by Mr. P. Riddell of Broken Hill.

The weather at this time of the year was characteristic of Central Australia,—warm, sunny days, with perfectly clear skies, and clear, pleasantly cool nights. The telescopic definition was in general excellent. The mean maximum shade temperature during September was 82°·2, and mean minimum 51°·5 F. Occasionally a change in the weather would bring up a considerable amount of dust from the dry gibber plains; and, in fact, the all-pervading fine dust and absence of facilities for photographic development made it necessary to take the plates back to Adelaide after the eclipse for development there.

4. *The Eclipse.*—The eclipse took place in the afternoon of September 21, totality lasting for 3 minutes 52 seconds, viz. from $3^h\ 33^m\ 17^s$ p.m. to $3^h\ 37^m\ 9^s$, Adelaide standard time, or G.M.T. $18^h\ 3^m\ 17^s$ to $18^h\ 7^m\ 9^s$ (Greenwich date, September 20).

As a result of our examination of Mr. Kennedy's photographs, Professor Grant and I decided to give the following exposures:—Plate I., 20 seconds on eclipse and check fields; Plate II., 30 seconds on eclipse and check fields; Plate III., 60 seconds on eclipse field only; Plate IV., 25 seconds on eclipse field only. This programme was closely adhered to, the only changes being in Plates III. and IV., which had 55 seconds and 20 seconds exposure respectively, owing to a slight uncertainty in the time counting and totality ending earlier than expected. The actual exposures were:—

		h m s		h m s	Exp s	
Plate I. G.M.T.		18 3 22	to	18 3 42	20	Eclipse field.
		18 3 52	,,	18 4 12	20	Check ,,
,, II.		18 4 22	,,	18 4 52	30	Check ,,
		18 5 2	,,	18 5 32	30	Eclipse ,,
,, III.		18 5 42	,,	18 6 37	55	Eclipse ,,
,, IV.		18 6 47	,,	18 7 7	20	Eclipse ,,

The first slide was drawn on the word "Go," and 5 seconds were allowed for vibration of the camera to cease before the exposure was made. The time between successive exposures, 10 seconds, was found sufficient to make the necessary changes and to give time for any vibration to cease. An additional precaution taken against instrumental vibration was the use of an exposure screen quite independent of the camera, as it was found that the shutter attached to the camera

in front of the object-glass, could not be operated without danger of setting up tremors which usually persisted for some little time, and affected the star images. The use of an exposure screen independent of the camera, and operated by means of a cord by the observer at the guiding telescope, was a decided improvement, and, I think, is to be recommended for eclipse work. As the camera could only be moved in right ascension, the Check field chosen was one 24·4 minutes in right ascension east of the Sun. An excellent stop device, suggested by Mr. K. Dixon, made the movement from eclipse field to check field, and *vice versa*, a quick and simple procedure. It was considered better to rate the driving clock as accurately as possible, and trust entirely to it, rather than attempt to control the movement of the camera by the slow-motion cord, a difficult matter during the hurried moments of the eclipse. The rate of the clock when adjusted for the Eclipse field stars, did not quite agree with the movement required for the Check field; consequently some slight elongation of star images in right ascension is noticeable, but the effect of this, so far as it affects stars uniformly over the whole plate, is eliminated in the equations.

A few days before the eclipse a change in the weather occurred, and on the morning of the eclipse, though conditions had improved and the dust had settled, there was a strong wind which threatened difficulty, as it was impossible to provide fully adequate wind shelter for the instruments. Fortunately the wind calmed down before the afternoon, and during totality it was almost perfectly calm. The eclipse took place in a beautifully clear sky, the Sun being at an altitude of 32°. The corona was of the minimum sunspot type, with one streamer extending to about two solar diameters above the Sun, and two similar streamers below. The corona and chromosphere were of moderate brightness, and there was only one fairly large prominence, which was situated on the south-west quadrant of the Sun, others on the upper limb being much smaller and inconspicuous. After the eclipse the plates were packed up and taken to Adelaide for development. The journey by motor car from Cordillo Downs *via* the south-eastern route to Broken Hill was a rough one, but was of great interest, as it traversed country made famous by the exploring expeditions of Captain Sturt and of Burke and Wills.

5. *Eclipse Plates.*—The plates reached Adelaide without mishap and were developed at once. A developing formula, suggested by Dr. Trumpler, giving good contrast, was used. This was Parkhurst's hydrokinone-potash developer for stellar photographs.

Solution A.	2¼	oz. hydrokinone.
	2	,, dry sodium sulphite.
	¼	,, sulphuric acid.
	64	,, water.
Solution B.	8	oz. potassium carbonate.
	2	,, dry sodium sulphite.
	½	,, potassium bromide.
	64	,, water.

Equal quantities of A. and B. were used.

The normal time of development is 10 minutes at 70° F., but the eclipse plates were given 12 minutes development in each case. Plates I. and II. show a much greater density of sky fog, due to the double exposure, than was expected. This has accentuated the density of the Corona on these plates. A fair number of stars both of the Eclipse field and of the Check field is shown on these two plates. Plates III. and IV., as previously mentioned, have only the stars of the Eclipse field.

6. *The Comparison Plates.*—These include (*a*) some of those taken by Mr. Kennedy in 1922 August, and (*b*) a series taken by myself during a return visit to Cordillo Downs six months after the eclipse, in 1923 March, when the sun was on the opposite side of the sky. These were taken in the early mornings, when the Eclipse field was at the same altitude and on the same side of the sky as during the eclipse. The Allegheny camera was taken off its bearings after the eclipse, and placed in the storeroom at the homestead, where it was left undisturbed until used for the comparison plates in 1923 March. These were brought to Adelaide and developed. The stars of both fields were identified, and an attempt was made by Professor Grant and myself to measure them at Adelaide, but owing to the only measuring machine available being found to be not sufficiently accurate for this purpose, we availed ourselves of the kind offer of the Astronomer Royal to measure them at Greenwich Observatory. They were sent in two shipments and reached England safely. An excellent idea which Professor Grant suggested was tried in the preliminary measurements at Adelaide, *i.e.* the ruling of double lines on the scale plate with a pair of diamonds suitably mounted. With this method each star appears inside a small square formed by the intersection of the pairs of cross lines in the two co-ordinates. It seems probable that the accuracy of measurement is increased by this method, as each star is referred to 4 fiducial lines instead of 2, and the idea is well worth trying.

Eclipse and Comparison Plates taken at Cordillo Downs.

Long. $9^h\ 22^m\ 32^s$ E. Lat. 26° 43′ S.

Eclipse Plates.

No.	Date and G.M.T.	Exp.	Bar.	Therm.	Field.
	d h m s	s	m	°	
I.	1922 Sept. 20 18 3 32	20	29·72	67·0	Eclipse.
	18 4 2	20			Comparison.
II.	,, ,, 20 18 4 37	30	,,	,,	Comparison.
	18 5 17	30			Eclipse.
III.	,, ,, 20 18 6 10	55	,,	,,	Eclipse.
IV.	,, ,, 20 18 6 57	20	,,	,,	Eclipse.

Notes.

I. and II. taken in the brass plate-holders belonging to camera.
III. and IV. taken in improvised wooden plate-holders.

Comparison Plates.

No.	Date and G.M.T.						Exp.	Bar.	Therm.	Field.
			d	h	m	s	s	m	°	
XII.	1922	Aug.	12	21	21	0	30	29·92	60 0	Eclipse.
XIV.	,,	,,	13	21	19	0	30	29·82	62·0	Eclipse.
4	1923	Mar.	14	7	31	16	90	29·57	71·0	Eclipse.
				7	36	15	90			Comparison.
9	,,	,,	18	6	53	55	60	25·59	67·0	Eclipse.
10	,,	,,	18	6	59	24	60	,,	,,	Eclipse.
5	,,	,,	18	7	4	23	60	,,	,,	Eclipse.
				7	11	22	60			Comparison.
8	,,	,,	18	7	15	51	60	,,	,,	Eclipse.
				7	18	21	60			Comparison.
11	,,	,,	19	6	42	18	60	29·63	69·0	Eclipse.
				6	46	18	60			Comparison.
12	,,	,,	19	6	54	16	60	,,	,,	Eclipse.
				6	58	15	60			Comparison.

Notes.

The Comparison field is the same declination and approximately 24·5 minutes east of the Eclipse field.

XII., XIV.—Conditions excellent, sky clear, no wind, dust, or smoke.

9, 10, 5, 8.—Clear, calm, light breeze at end.

11, 12.—Clear, calm, excellent conditions.

II. *Measurement and Reduction of the Plates.* By C. R. Davidson.

1. The great difficulty in the discussion of photographs taken during the eclipse, is the accurate separation of the scale value from the Einstein effect. Each produces a radial distortion of the photograph varying as the distance in the former case, and as the inverse distance in the gravitational effect. If there are a sufficient number of stars well distributed from the limb outwards to a degree or more from the sun, these quantities may be separated readily. At a distance of 15′ from the sun's limb the gravitational displacement is ″·87, and at 45′ it is ″·44. If the majority of the stars fall outside the higher limit, the difficulty of separating scale from gravitational effect is very great, and would require a very small accidental error in the measurement and still more in the position of the images on the plates.

If, however, the scale value can be determined independently, then a measure of the distance between two stars on opposite sides of the sun will give a determination of the gravitational effect. For example, two stars on opposite sides of the sun at distances as great as one degree from the centre, will show a displacement of ″·88 in their distance apart.

2. The scale of the plate may be determined by taking a photograph of the Eclipse field of stars, under as nearly as possible the same conditions, some months before or after the eclipse. If the constancy of the scale could be relied upon, this would be sufficient. A check on this constancy is obtainable by taking on the same plate photographs

of a second field at the time when these comparison plates are taken, and by exposing the plates taken during the eclipse to this second field on the night following. The criticism which would be made on this procedure is that the scale may be different under the conditions of the eclipse, and on the succeeding night.

3. This difficulty can be met by photographing, *during the eclipse*, a field of stars, a few degrees from the sun, on the plate which has been already exposed to the Eclipse field. When this is compared with a similar plate, taken some months after the eclipse, the Comparison field gives a determination of any change of scale value. This is utilised for the plates of the Eclipse field and leaves the Einstein effect alone to be determined.

Thus if E denotes the Eclipse field of stars
and c ,, Comparison ,, ,,
E and c are taken on the same plate during the eclipse.

Similarly, E_0 and c_0 represent these two fields taken some months after the eclipse.

A comparison of c and c_0 gives the change of scale. This is utilised in the comparison of E and E_0 which then gives the gravitational effect.

4. During the eclipse four photographs were taken:—

I. and II. of the Eclipse field, together with the Comparison field. III. and IV. of the Eclipse field only.

No. IV. showed too few stars to yield a result, but I., II., and III. were measured. It was at once evident that for some reason the scale of III. was different from that of Nos. I. and II., and as no Comparison field had been impressed on this plate, no further use could be made of it. The difference is probably due to the fact that this plate was exposed in an improvised wooden carrier. An error of ·02 inches in register would account for the difference of scale.

Plates I. and II. were measured very thoroughly, and the final result depends on these two plates alone. In all, nine comparison plates have been measured. Of these, XII. and XIV. of the Eclipse field only, were taken on August 12 and 13 preceding the eclipse. The remaining seven were taken in the special expedition to Cordillo Downs in the March following the eclipse, viz. two plates, Nos. 9 and 10, of the Eclipse field only, and five plates, Nos. 4, 5, 8, 11, 12, of both Eclipse and Comparison fields. The final determination of scale value depends on these five plates, the other four photographs, however, have been of secondary use in the measurement.

5. Measurement was effected by the same method as that used at Greenwich in determination of stellar parallaxes. Short scale divisions are ruled with a diamond on a glass plate in the positions of the stars it is desired to measure. This plate is superposed on each photograph successively, and the star images measured relatively to their corresponding rulings, the image and ruling being viewed together in the microscope; the quantity measured never amounting to more than a few seconds of arc. Whatever errors are made in ruling the divisions disappear in the differential comparison of one plate with another. This is strictly true when *all* the stars are measured on *all* the plates, but if

on any plate a star is missing, it is convenient to apply a correction for "division error" obtained from the photographs containing all the stars, to avoid the necessity of a re-reduction of the plates. A correction of this description was applied to the measures of the Eclipse field on Plate I., three stars being found too faint for measurement. After careful examination of the plates it was thought that there were possibly nineteen measurable stars in the Eclipse field and seventeen in the Comparison field. Two glass plates were ruled accordingly with divisions in the positions of these stars, one for the Eclipse field and one for the Comparison field. These will be referred to hereafter as glass scales. Under the microscope, some of the stars were found to be too indefinite to admit of satisfactory measurement, and ultimately eleven stars were retained in the Eclipse field on Plate I., and fourteen stars in the Eclipse field on Plate II., together with sixteen stars in the Comparison field on both plates.

These stars are shown in the following tables:—

Stars in the Eclipse Field.

Centre R.A. $11^h\ 50^m\ 40^s$. Dec. $+1°\ 0'$.

Serial No.	B.D. No.	Mag.	Co-ordinates. x. Int.	Co-ordinates. y. Int.	Gravitational Factor. Ex.	Gravitational Factor. Ey.	Displacement. x.	Displacement. y.
Sun	°		0·00	0·00			"	"
1	0 2831	7·6	−13·72	− 6·16	−·061	−·027	−·16	−·07
2	0 2843	6·5	− 8·00	− 5·04	−·089	−·056	−·23	−·15
3	3 2560	7·7	− 6·95	+ 9·49	−·050	+·069	−·13	+·18
4	β Virg.	3·7	− 5·77	+ 6.87	−·072	+·085	−·19	+·22
5	−0 2510	8·0	+ 0·27	−11·45	+·002	−·087	+·01	−·23
6	4 2544	8·0	+ 0·65	+15·12	+·003	+·066	+·01	+·17
7	1 2628	8·0	+ 1·05	+ 3·02	+·103	+·297	+·26	+·75
8	1 2633	7·7	+ 5·06	+ 1·74	+·177	+·061	+·47	+·16
9	1 2636	6·8	+ 6·26	− 0·20	+·160	−·005	+·42	−·01
10	2 2499	7·1	+ 6·74	+ 7·19	+·069	+·074	+·18	+·20
11	−1 2600	7·7	+ 6·99	−14·15	+·028	−·056	+·07	−·15
12	−0 2520	6·8	+ 9·07	−13·29	+·035	−·051	+·09	−·13
13	0 2881	8·3	+10·38	− 2·64	+·090	−·023	+·24	−·06
14	2 2509	8·3	+13·62	+ 5·12	+·064	+·024	+·17	+·06

Stars in the Comparison Field.

Centre R.A. $12^h\ 15^m\ 10^s$. Dec. $+1°\ 0'$.

Seria No.	B.D. No.	Mag.	x. Int.	y. Int.	Serial No.	B.D. No.	Mag.	x. Int.	y. Int.
	°					°			
1	0 2902	7·7	−13·87	− 7·53	9	0 2920	6·3	− 0·70	− 7·70
2	2 2517	6·6	−13·47	+ 7·61	10	η Virg.	4·0	+ 1·07	− 7·02
3	4 2585	8·1	− 9·45	+16·16	11	4 2604	5·5	+ 1·74	+15·64
4	−0 2549	8·0	− 7·90	−10·26	12	4 2609	8·4	+ 4·47	+15·54
5	3 2616	7·5	7·42	+ 9·65	13	2 2534	8·8	+ 6·38	+ 5·43
6	−0 2554	7·5	− 5·92	−10·77	14	0 2942	8·2	+ 9·28	− 4·84
7	2 2526	8·0	− 1·70	+ 5·74	15	2 2539	7·3	+ 9·76	+ 8·41
8	3 2626	8·2	− 1·19	+14·43	16	0 2944	7·7	+10·82	− 4·27

The stars in both the Eclipse and Comparison fields on the nine comparison plates taken in the months before and after the eclipse were measured relatively to the glass scale. As these photographs were taken at an altitude of less than 30°, corrections were applied for the second order terms of refraction, and proper motions also were applied to bring these and the plates taken during the eclipse to a common epoch.

Equations of the form,

$$\mathrm{D}x = ax + by + c$$
$$\mathrm{D}y = dx + ey + f$$

were then obtained for each star, where x and y are the co-ordinates expressed in units of one réseau interval and Dx Dy are measures relative to the glass scale expressed in seconds of arc.

The solution of these equations gives a and e—the scale of the photographs in the two co-ordinates (after correction for refraction and aberration).

The scale values of the Eclipse and Comparison fields derived from each plate are given in the following tables:—

Scale Value of Eclipse and Comparison Fields taken 1923 *March.*

Eclipse Field.

No.	a.	Refr.	Aberr.	a_0 or Scale.	e.	Refr.	Aberr.	e_0 or Scale.
4	−·300	+1·175	+·004	+·879	−·024	+·478	+·004	+·458
5	−·096	·946	+·001	·851	−·016	+·420	+ 1	·405
8	−·290	1·190	1	·901	−·081	·483	+ 1	·403
11	+·159	·696	−·001	·854	+·064	·356	−·001	·419
12	+·013	·846	−·001	·858	−·050	·394	−·001	·344

Comparison Field.

4	−·317	+·811	+·011	+·505	+·054	+·384	+·011	+·449
5	−·203	·703	·006	·506	+·041	·357	·006	·404
8	−·262	·786	·006	·530	−·069	·378	·006	·315
11	+·021	·523	·006	·550	+·056	·310	·006	·372
12	−·106	·613	·006	·513	−·045	·333	·006	·294

Eclipse − Comparison.

	+·374	+·009
	·345	·001
	·371	·088
	·304	·047
	·345	·050
Mean	+·348	+·039

The plates taken during the eclipse were reduced in a similar manner.

The Comparison field on these plates being sufficiently far from the sun will not be distorted appreciably by gravitation, and consequently serve to give the true scale value during the eclipse.

The scale constants a_0, e_0 in the preceding table are arbitrary quantities depending on the assumed positions of the stars adopted when ruling the glass scales. Consequently the quantities a_0, e_0 for the Eclipse field compared with its glass scale are not the same as those for the Comparison field compared with its corresponding glass scale. Their differences, however, should be constant.

This difference is shown in the columns Eclipse – Comparison, and when applied to the values a_0 and e_0 found for the *Comparison field* on the eclipse plates gives the values of a_0 and e_0 for the *Eclipse field* on the eclipse plates, from which, by applying refraction, the values of a and e are found.

From the Comparison field taken during the eclipse we obtain the following values of a and e, corrected to a_0 and e_0, by the application of refraction and aberration.

Comparison Field.

	a.	Refr.	Aberr.	a_0.	e.	Refr.	Aberr.	e_0.
Ec. I.	+·108	+·375	−·006	+·477	+·136	+·272	+·006	+·402
,, II.	·098	·380	−·006	+·472	+·056	+·272	−·006	+·322

Applying the differences found above for eclipse minus comparison

$$Da_0 = +·348 \text{ and } De_0 = +·039,$$

we find for the eclipse plates themselves the following values of a_0 and e_0 and, including refraction, etc., a and e.

Eclipse Field.

	a_0.	Refr.	Aberr.	a.	e_0.	Refr.	Aberr.	e.
Ec. I.	+·825	+·485	0	+·340	+·441	+·300	0	+·141
,, II.	·820	·492	0	·328	·361	·302	0	·059

The values $a = +·340$ $e = +·141$ for Ec. I.

and $a = +·328$ $e = +·059$,, II.

were adopted for those constants on the Eclipse field.

The measures of the Eclipse field were now connected with the gravitational displacement by the equations

$$Dx = c + ax + by + \alpha E(x)$$
$$Dy = f + dx + ey + \alpha E(y),$$

where $E(x)$ and $E(y)$ are the coefficients of the gravitational displacement.

After substituting the adopted values of a and e the values of the other constants were then determined.

The unit in which the co-ordinates are expressed is one réseau interval of five millimetres which on these plates represents 633 seconds

of arc. The constant a then would be the gravitational displacement at 633 seconds of arc from the sun's centre.

The value of a thus obtained from the photographs is :—

	x Co-ord.	y Co-ord.	Mean.
	″	″	″
Plate I.	3·49	3·63	3·56
,, II.	2·46	1·07	1·76

corresponding to a displacement at the limb of

	x.	y.	Mean.
	″	″	″
Plate I.	2·31	2·40	2·36
,, II.	1·64	0·71	1·18

General Mean 1″·77.

In this reduction no account was taken of the effect of the Sun's gravitational field on the positions of the stars on the comparison plates taken during the eclipse. This would have produced a slight contraction of the scale, which would necessitate corrections to the constants given above amounting to +·002 in a and −·002 in e. When these corrections are carried into the equations for the Eclipse field they produce alterations in the value of the Einstein constant of amount −0″·08 in x and +0″·08 in y, leaving the final result unchanged.

Although the mean result is in satisfactory agreement with the predicted value it will be observed that the separate results show considerable discordance, and this appears also in the residuals of the individual stars. The probable error of the final result may be taken as ±0″·3. It should be pointed out that the observations were attended by very considerable difficulty, that the focal length of the telescope was short, and that the altitude of the sun (30°) was low for observations of this refined character.

Micrometrical Measures of Double Stars and New Double Stars.
By Rev. T. E. Espin and W. Milburn.

In the following lists stars marked E were measured with the 24-inch reflector, and M with the 17¼-inch reflector.

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.	Obs.
	h m	° ′	°	″				
Espin 115	0 17·4	+61 41	82·5	9·30	8·0 10·0	4	·826 AB	M
			80·3	46·55	D=9·5	3	·819 AD	M
			54·8	24·55	C=10·5	3	·835 AC	M
β 780	26·7	37 12	135·2	2·83	8·5 10·5	3	·980	E
h 1033	27·5	62 45	43·3	6·71	10·0 10·1	2	·871	M
h 627	40·6	35 53	163·4	8·37	10·4 11·2	2	·836	E

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.	Obs.
	h m	° ′	°	″				
Espin 1867	0 40·7	60 59	270·5	6·45	9·0 10·5	2	·945	AB M
			298·7	64·56	C=9·2	1	·928	AC M
Espin 225	42·8	38 6	240·7	7·95	9·3 14·0	2	·394	E
h 1070	1 0·2	61 41	51·8	3·86	11·0 11·5	3	·851	AB M
			90·0	9·24	C=12·5	6	·881	AC M
			18·6	49·88	D=10·0	2	·900	AD M
			100·6	11·61	E=11·0	2	·900	DE M
			65·5	6·45	F=13·5	3	·904	EF M
h 1087	33·4	38 30	70·1	11·66	9·0 9·4	3	·926	E
Σ 141	34·2	38 28	298·3	1·54	7·2 7·6	3	·926	E
Espin 158	43·3	40 28	49·5	7·29	8·2 9·8	2	·963	E
Espin 162	51·7	36 17	208·2	11·49	8·2 10·8	2	·966	E
Espin 163	52·1	36 17	21·0	5·45	9·5 9·6	2	·966	E
Tucker	2 11·8	37 38	238·7	2·67	8·8 10·5	2	·990	E
Σ 373 rej.	3 13·9	62 22	118·1	20·62	7·0 9·5	2	·990	M
Espin 165	29·5	63 53	174·1	3·89	9·5 11·2	2	·990	M
OΣΣ 36	31·2	63 34	72·2	46·87	6·5 8·0	2	·990	M
h 3305	7 54·3	37 10	228·4	4·43	9·4 9·7	2	·198	E
Furner	17 31·0	39 8	54·8	1·96	9·1 9·4	2	·583	E
Espin 241	18 48·0	36 41	71·3	2·24	9·5 10·5	4	·827	E
Σ 2423	51·0	65 6	195·9	2·86	8·5 9·8	3	·629	M
Lewis	51·5	36 54	31·6	3·83	10·5 11·5	4	·757	E
Espin 242	19 47·4	36 28	31·4	2·00	9·6 9·9	2	·869	E
Scheiner 717	50·0	38 43	104·0	4·39	10·5 11·0	2	·657	E
Scheiner 750	54·1	38 25	28·9	6·93	10·1 11·0	2	·708	E
Scheiner 791	57·5	38 56	57·6	5·64	10·0 11·4	2	·708	E
Scheiner 825	59·7	37 50	149·8	6·68	8·9 10·5	3	·777	E
Scheiner 1025	20 9·9	38 54	166·0	5·25	10·6 10·7	2	·671	E
Ho. 590	13·0	39 19	208·6	3·27	8·6 10·5	3	·638	E
Scheiner 1079	15·2	38 50	107·2	6·02	10·7 11·3	2	·691	E
Scheiner 1088	16·6	38 21	240·0	4·10	9·4 9·7	2	·765	E
Espin 206	25·1	37 50	119·1	4·01	9·0 9·2	3	·878	E
Espin 245	26·5	39 47	156·6	4·25	9·5 9·6	3	·878	E
Espin 247	32·6	36 30	146·5	5·35	9·4 11·0	2	·869	E
A.G. 261	38·2	38 6	71·1	1·96	9·1 9·2	2	·749	E
Espin 250	48·2	36 22	82·7	4·66	9·5 12·0	3	·897	E

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.	Obs.
	h m	° ′	°	″				
h 1596	20 51·8	38 39	291·9	16·92	9·0 9·8	3	·864	E
Barnard 14	55·9	37 29	246·5	1·05	9·2 9·3	3	·890	E
Lv. 9	58·3	38 50	189·8	2·47	9·4 10·0	2	·647	E
Espin 252	21 6·2	37 0	172·5	2·70	9·3 9·5	1	·964	E
Espin 253	6·7	37 12	31·7	2·50	9·1 9·3	3	·954	E
Espin 207	7·2	37 56	154·9	2·00	9·2 9·3	3	·954	E
Espin 257	9·1	38 32	322·3	5·79	8·2 12·0	2	·938	E
Espin 208	9·3	37 9	141·2	3·83	9·5 10·5	3	·927	E
Scheiner 1458	20·1	37 15	310·9	7·41	9·2 10·2	2	·793	E
Espin 258	25·5	35 59	359·1	5·81	9·5 11·5	2	·912 BC	E
			20·5	23·55	A=9·4	2	·912 AB	E
Scheiner 1521	26·3	36 35	222·9	2·95	9·5 10·4	3	·959	E
Σ 2806	27·4	70 7	250·6	13·43	3·0 8·0	3	·760	M
A 1444	35·7	37 22	277·6	1·20	9·1 9·2	5	·782	E
h 1742	22 6·4	67 14	332·6	24·03	8·5 9·5	3	·665	M
h 968	35·0	36 22	107·2	4·21	8·5 9·2	2	·990	E
Σ 2947	45·6	68 2	65·6	3·74	7·2 7·3	2	·793	M
OΣ 530	46·0	67 38	212·3	5·18	10·0 10·5	2	·798	M
h 5531	23 4·0	35 55	63·8	8·47	10·0 10·5	4	·903	E
h 1865	11·3	67 14	215·4	7·44	10·0 10·5	2	·962	M
Espin 219	13·8	35 48	311·9	4·74	9·5 9·6	3	·946	E
Espin 267	25·8	38 57	169·3	2·59	9·6 11·0	3	·905	E
Espin 269	44·0	40 46	218·4	7·03	8·5 12·5	2	·954 AB	E
			135·6	15·27	C=12·0	3	·957 AC	E

Notes.

h 1070, $1^h\ 0^m{\cdot}2$. A star of mag. 11·0 follows E, and another of mag. 11·5 is midway between A and F. A third star of mag. 14·5 was suspected preceding A, and a little closer to it than C.

Σ 2423. Distance probably too large.

Lewis, $18^h\ 51^m{\cdot}5$. *f* star of a little triangle.

Scheiner 717, $19^h\ 50^m{\cdot}0$. *Comes*, 14·0 mag., P. 318°·3, D. 21″·6.

Barnard 14, $20^h\ 55^m{\cdot}9$. Not well defined on any of the three nights. The *comes* 15 mag. seen.

Scheiner 1458, $21^h\ 20^m{\cdot}1$. Scheiner's place is 10^m too small in R.A.

h 5531, $23^h\ 4^m{\cdot}0$. *h*'s place, 1^m too great in R.A.

Double Stars not found.

O. Stone, $19^h\ 35^m \pm$. +37° 55′. Not found, nor by β.

Scheiner 767, $19^h\ 55^m$. +38° 15′ (1900). There is no *comes* at P. 328°, D. 3″·1.

Scheiner 1532, $21^h\ 37^m$. +37° 50′ (1900). I failed to find this double. Espin 2066 was measured in looking for it.

Scheiner 1221, $20^h\ 38^m$. +39° 35′ (1900). Not found on two nights, 1922.

New Double Stars.

The following double stars have been found and measured with the 24-inch Calver reflector.

E.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.
		h m	° ′	°	″			
	+35°46	0 13·4	+35 26	90°8	14·20	7·5 11·2	2	·838
2007	+35·75	22·4	36 7	33·6	3·62	10·5 10·7	2	·347 BC
				60·7	29·13	A=9·3	3	·540 AB
2008		55·0	36 46	42·3	2·02	10·5 11·0	4	·631
2009		1 22·8	36 37	279·8	4·46	10·0 10·5	2	·909
2010	+37·478	59·8	37 31	172·9	7·13	9·1 12·0	3	·580
2011		2 8·4	37 6	345·7	4·04	10·2 11·7	3	·981
	+38·461	14·9	38 27	91·3	10·70	9·1 9·2	2	·448
	+37·572	25·6	37 51	330·4	19·86	7·2 11·7	2	·883
2012	+38·1915	8 23·7	38 35	88·3	5·58	9·7 10·3	2	·198
2013		45·0	38 46	62·8	4·16	10·2 10·3	1	·224
2014	+38·2938	17 24·1	38 52	88·6	1·81	9·2 11·7	3	·606
2015	+39·3166	32·2	39 36	29·0	4·87	9·7 10·9	3	·582
2016	+37·2940	41·5	37 54	217·3	4·04	7·9 12·0	2	·657
2017	+38·3078	18 3·0	38 37	260·3	2·22	9·1 12·0	3	·601
2018	+39·3325	4·4	39 4	238·5	5·46	8·9 9·6	2	·563
	+37·3060	12·3	37 21	80·3	9·72	9·0 9·1	2	·671
2019	+37·3181	34·2	38 1	115·7	6·69	9·1 11·5	2	·616
2020	+37·3226	41·7	37 50	347·1	2·91	9·3 11·0	2	·608
2021	+38·3294	43·2	38 46	306·5	3·58	10·5 11·7	2	·563 BC
				252·4	21·32	A=9·6	3	·569
2022	+36·3267	44·0	36 40	316·3	4·31	9·4 11·0	2	·765
2023	+36·3271	44·9	36 5	249·3	6·26	8·7 12·0	2	·810
2024	+36·3285	46·9	36 7	355·3	7·22	9·4 13·0	3	·817
2025	+37·3259	48·2	37 32	82·5	4·01	10·2 13·9	3	·712 BC
				11·8	5·60	D=10·7	2	·707 BD
				324·0	19·68	A=9·5	3	·712 AB
2026	+37·3262	48·6	37 24	339·9	5·05	13·2 13·7	2	·712 BC
				107·9	19·74	A=7·5	3	·732 AB
2027	+37·3272	50·2	37 44	138·8	3·87	9·1 13·0	2	·704
2028	δ^2 Lyræ	51·0	36 46	137·7	2·18	11·2 11·6	2	·793 BC
				350·2	87±	A=4·5	2	·793 AB
2029	+35·3405	51·3	36 3	209·4	5·35	9·2 9·6	2	·823
2030	+36·3340	53·7	36 56	217·3	4·08	9·5 12·5	2	·753 AB
				231·1	13·59	C=10·5	2	·753 AC
2031	+35·3431	54·8	35 46	40·4	5·78	9·5 10·8	2	·842

E.	B.D.	R.A. (1900). h m	Decl. ° ′	P. °	D. ″	Mags.	Nts.	Date 1923.
2032	+36°3353	18 55·5	36 46	217·6	4·15	9·4 12·2	4	·770
2033	+37°3314	58·6	37 52	262·4	4·83	9·3 9·5	3	·682
	+37°3324	19 0·1	37 59	299.5	11·81	8·5 10·2	2	·630
2034	+38°3440	4·0	38 41	73·5	5·02	9·5 14·0	3	·602
2035	+38°3457	6·3	38 39	176·0	3·74	9·4 10·5	2	·652
2036	+37°3467	24·2	37 52	148·7	3·13	9·6 11·5	3	·721
2037		24·6	37 15	40·4	5·18	9·3 12·5	4	·813
2038	+37°3488	27·2	37 44	219·7	4·92	9·5 12·5	3	·736
2039	+38°3637	28·4	38 16	276·0	1·64	9·3 9·4	3	·638
2040		35·7	36 53	108·4	1·60	10·5 11·0	3	·851
2041	+36°3665	37·7	36 54	341·6	6·03	9 3 11·0	2	·842
2042	+38°3804	50·0	38 11	84·6	6·93	9·5 13·0	2	·630
2043	+37°3686	52·9	37 20	114·2	1·47	10·0 10·5	2	·853
2044	+38°3872	58·6	38 32	26·4	6·37	9·5 13·7	2	·657
2045		59·4	37 26	129·4	2·72	10·5 11·0	3	·832
2046	+38°3933	20 7·1	38 42	99·2	5·98	9·1 12·7	3	·767
2047	+38°3965	11·0	38 37	197·3	4·35	10·1 10·3	2	·845
				174.9	32·13	A=9·0	2	·845
2048		12·0	38 51	259·7	3·08	9·1 10·6	3	·733
2049		14·6	38 6	265·4	3·14	10·5 10·5	2	·853
2050	+38°4020	15·6	39 2	92·5	5·98	9·2 12·7	2	·693
2051	+39°4134	15·6	39 19	302·9	4·91	9·5 12·0	3	·615
2052		16·5	38 19	147·9	2·07	10·6 11·2	4	·774
2053		39·2	38 28	216·8	4·04	10·2 11·7	3	·733
2054	+37°4054	43·2	38 6	188·9	2·96	9·5 12·2	2	·765
2055	+38°4286	52·4	38 37	303·4	6·36	9·0 12·5	2	·845
2056		53·9	38 25	78·9	2·21	9·7 12·5	3	·682
2057	+38°4315	56·5	38 22	254·2	4·43	9·4 11·2	3	·733
2058		58·5	37 29	308·7	3·39	10·7 10·7	3	·858
2059	+38°4351	21 3·1	38 15	265·9	6·48	9·5 12·0	2	·657
2060		3·7	37 33	267·4	3·93	10·2 11·1	2	·798 AB
				271·8	17·66	C=11·2	2	·798 AC
2061		11·2	38 8	297·7	2·79	9·6 11·5	3	·769
2062	+37°4273	15·9	37 50	324·9	4·41	9·4 13·0	2	·780
2063		27·9	37 44	226·9	2·68	10·0 10·2	3	·750
2064	+37°4338	28·0	38 7	246·7	11·89	9·0 9·5	2	·693
2065	+36°4620	33·5	36 45	185·1	4·97	9·5 9·6	4	·849
2066		35 8	37 46	224·6	5·46	10·2 10·5	2	·657
2067	+36°4639	36·4	36 53	226·1	5·28	9·5 13·5	3	·863

E.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.
	°	h m	° ′	°	″			
2068		21 49·6	35 42	346·1	2·41	10·1 10·5	2	·957
2069	+36·4713	52·2	36 32	111·8	4·22	9·5 13·0	3	·800 AB
				217·9	6·83	C=11·0	2	·793
2070		22 18·9	36 12	152·3	3·93	10·6 11·2	3	·758
2071	+36·4842	24·0	36 57	29·5	4·31	12·0 15·0	2	·734 BC
				262·7	26·87	A=7·5	2	·734 AB
2072		26·2	36 58	264·8	1·58	9·8 10·5	3	·783 BC
				314·5	13·16	A=9·7	3	·783 AB
2073	+35·4838	29·7	35 27	209·2	7·45	9·1 9·4	3	·954
2074	+36·4889	33·1	36 30	80·3	2·03	9·4 11·5	4	·826
2075	+35·4902	48·0	36 1	269·4	5·70	9·5 10·5	3	·818
	+35·4913	50·8	36 8	101·3	10·09	9·2 10·0	2	·813
2076		55·9	36 43	169·5	1·74	9·5 9·5	3	·773
2077	+35·4944	57·2	36 10	152·6	5·62	9·3 12·5	3	·870
2078	+35·4962	23 3·9	35 48	300·3	7·09	9·5 12·5	4	·938

Notes.

No. 2008. At P. 230° from B.D. +36°·176.
2019. *Comes*, 12·5 mag., P. 26°·8, D. 16″·4.
2028. δ^2 Lyræ. Mr. Mervyn Ellison at Armagh kindly measured AB on 1923, November 14. P. 348°·9, D. 86″·2.
2047. *Comes* to A, 13.5 mag., P. 99°·6, D. 11″·24: another more distant in same direction.

New Double Stars.

The following double stars have been found and measured with the 17¼-inch Calver reflector.

M.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.
	°	h m	° ′	°	″			
276		0 4·1	+63 2	269·3	3·42	11·0 12·0	3	·932
277	+63·49	21·4	63 57	71·7	6·49	9·1 9·6	2	·765
278		22·8	63 35	23·7	3·87	11·0 12·0	5	·812
279		33·4	61 34	40·1	3·53	9·5 11·5	3	·977
	+61·225	1 6·4	62 7	220·0	8·74	9·5 11·0	5	·840 AB
				10·1	11·82	C=11·5	4	·856 AC
	+62·274	28·6	62 34	79·0	58·39	6·5 11·0	3	·910 AB
280				45·4	6·36	C=11·5	2	·905 BC
	+61·390	2 5·5	61 54	253·7	67·41	9·2 11·0	2	·836 AB
281				143·6	5·18	C=11·1	2	·836 BC
282	+61·417	21·5	61 40	143·3	4·05	9·5 11·0	2	·871 AB
				44·2	4·13	C=10·0	3	·880 AC

M.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.
	°	h m	° ′	°	″			
283		2 24·4	62 10	264·9	3·55	11·0 12·0	2	·985
284	+61·447	31·8	62 0	324·7	4·93	8·5 11·0	4	·976
285		7 42·1	65 4	254·0	3·24	9·5 11·0	2	·198
286	+66·1127	18 44·6	66 43	287·1	3·09	9·1 12·0	2	·594
287		54·3	66 58	345·1	4·14	10·5 13·0	3	·631
		54·3	66 57	3·5	12·27	10·0 11·5	3	·631
288		19 10·5	66 50	287·7	3·36	10·0 12·0	3	·577
	+66·1175	12·8	66 25	287·0	34·32	9·5 10·0	3	·577 AB
289				2·3	4·60	C=11·5	3	·577 BC
	+67·1167	34·6	67 35	81·4	19·15	8·5 11·5	4	·626 AB
290				301·0	3·37	C=12·5	3	·622 BC
291		34·8	67 51	155·6	3·58	11·0 13·0	2	·671
	+67·1172	36·0	67 41	342·1	9·14	9·5 9·8	2	·657
292		39·5	68 1	311·2	6·17	10·0 11·5	4	·693
293	+68·1100	20 1·9	69 6	90·1	3·88	9·3 11·5	3	·751
294	+66·1268	2·5	67 4	158·4	2·75	9·5 11·0	3	·577
295		3·5	67 37	305·0	7·33	10·0 13·0	4	·721
296	+67·1236	13·4	67 18	286·5	2·60	9·5 12·0	3	·623
297	+66·1279	13·6	66 47	216·7	3·58	9·5 10·0	3	·577
298	+67·1240	17·8	67 24	156·9	9·23	9·5 9·7	2	·671
		33·1	67 40	357·6	8·14	10·0 12·5	3	·745
299	+67·1259	36·1	67 57	205·6	7·95	9·5 10·5	2	·610
300		40·4	66 48	16·0	2·80	9·5 12·5	9	·671
301		49·6	66 45	330·2	4·63	10·0 12·0	3	·848
302	+66·1333	50·4	66 38	323·4	4·98	9·1 9 2	2	·776
303		21 4·2	69 18	351·4	4·86	10·5 12·5	3	·712
304	+68·1203	18·2	69 3	48·3	4·55	9·0 11·5	3	·745
305	+68·1236	37·3	68 18	83·0	5·64	8·5 10·5	2	·646
		47·6	67 34	227·3	8·53	11·0 11·5	2	·637
306		55·5	67 49	142·6	4·01	11·0 11·0	3	·735
		55·6	66 53	196·1	65·25	10·0 12·0	1	·928 AB
307				156·9	3·89	C=12·5	2	·938 BC
		56·3	69 1	158·1	33·90	10·2 11·0	3	·865 AB
308				89·1	5·48	C=11·2	3	·865 BC
	+65·1697	56·9	65 51	117·7	11·00	9·5 10·0	2	·765
309	+65·1708	59·5	65 21	155·7	5·32	8·0 10·5	2	·840
	+67·1386	22 0·1	67 46	274·9	30·62	8·8 10·8	3	·865 AB
310				194·0	4·76	11·0 11·5	3	·865 BC
	+66·1470	0·3	66 47	312·2	18·80	8·0 9·2	2	·957

M.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1923.
	°	h m	° ′	°	″			
311		22 4·1	67 25	196·3	3·40	11·0 12·0	4	·736
	+66·1481	8·8	66 33	215·2	8·51	9·5 9·7	3	·769
312	+67·1431	17·9	67 58	6·1	5·02	9 5 10·0	3	·795 AB
				302·1	40·00	C=10·0	2	·793 AC
313	+66·1505	21·0	66 39	198·7	7·25	9·5 11·0	2	·728
314		25·1	66 48	166·1	4·31	10·0 11·5	2	·728
315	+67·1461	39·4	67 32	235·1	3·16	9·5 9·6	2	·728
316		39·5	68 4	121·0	3·53	10·0 10·1	5	·836 AB
				346·4	10·55	C=13·0	5	·852 AC
317		42·2	67 33	12·7	5·71	10·5 12·0	2	·728
318	+69·1319	23 11·7	69 17	248·2	2·89	8·5 8·7	4	·971 AB
				238·9	9·69	C=11·0	3	·959 AC
319		25·9	67 49	130·6	6·54	9·5 12·0	3	·737
320	+68·1382	30·9	69 4	341·5	5·25	9·5 11·0	2	·950
321		44·5	69 9	83·3	3·25	11·0 11·0	6	·801 AB
				254·3	6·39	C=13·0	9	·817 AC
322		48·9	68 39	54·1	4·01	11·0 12·5	2	·985

Notes.

No. 295. Only one measure of distance secured, and that is probably too large, as the estimate on discovery was $4'' - 5''\pm$.

300. Not sure of its duplicity on several nights. It certainly seems to have a *comes*, probably very faint, at $4''\pm$ and $0°\pm$. Another *comes*, more distant, was suspected on one occasion.

307. B.D. $+66°\cdot1465$ (9·0 mag.) is at $136°\pm$; $250''\pm$.

316. C is very faint and difficult to measure.

321. C is a very difficult object with this aperture.

Revised Elements of CG Cygni. By A. Stanley Williams.

R.A. $= 20^h\ 54^m\ 12^s$ Decl. $= +34°\ 47'\cdot4$ (1900).

The elements of variation of this star published in the *Monthly Notices*, 82, 303, were based on eleven minima derived from observations extending over a period of little more than three months, and they did not satisfy a photographic observation, dated twenty-one years earlier, which showed it relatively faint. Moreover, three of these minima were only roughly determined.

Three additional minima were observed in 1922 and six others in 1923, so that material now exists for deriving more exact elements. The three minima of 1922 fall almost midway in point of time between those of 1921 and 1923, so that they are practically of no use for correcting the period. The revised period has, therefore, been derived by comparing

the six minima observed in 1923 with eight of those determined in 1921. The three minima of 1921, only roughly determined, were rejected altogether. The epoch of minimum is based on all the seventeen well-observed minima. The revised elements of variation are:

$$\text{Heliocentric G.M.T. of principal minimum} = \text{J.D. } 2422967{\cdot}4283 + 0^{d}{\cdot}631143\ \text{E.}$$

The following table gives in column 2 the observed times of the principal minimum and in column 3 the computed times. The last column contains the differences O − C. The observations were treated as being all of equal weight.

E.	Observed Min.	Computed.	O − C. d
− 12116	J.D. [2415320·525]	·4997	[+0·0253]
+ 16	2422977·5298	·5266	+ ·0032
38	2991·4062	·4117	− ·0055
41	2993·3068	·3052	+ ·0016
52	3000·2430	·2477	− ·0047
60	3005·2957	·2969	− ·0012
117	3041·2750	·2720	+ ·0030
136	3053·2694	·2638	+ ·0056
155	3065·2590	·2555	+ ·0035
583	3335·3826	·3847	− ·0021
640	3371·3589	·3598	− ·0009
700	3409·2242	·2284	− ·0042
1174	3708·4013	·3902	+ ·0111
1177	3710·2854	·2836	+ ·0018
1185	3715·3277	·3327	− ·0050
1215	3734·2659	·2670	− ·0011
1253	3758·2485	·2505	− ·0020
1261	3763·2960	·2996	− ·0036

The first observation is not the actual minimum but the time of the early photographic observation previously alluded to. It might well have been half an hour before or after the actual minimum. The revised elements satisfy this observation within $0^{d}{\cdot}0253$ or 36^{m}, so that in the absence of knowledge of the photographic light-curve it is doubtful if any correction to the period is indicated. Also there is no definite indication of any departure from a uniform period.

The first observation of 1923 (E. 1174) is unusually discordant. The average residual (without regard to sign) is $\pm 5^{m}$, but the observed time was in this case 16^{m} late. The star was well observed both during the decreasing and the increasing phase, and I have not been able to find any sufficient reason for the discordance, or for rejecting the observation.

Polvarth, St. Mawes:
1923 *Dec.* 25.

X Herculis. By P. M. Ryves. (Plate 7.)

This star is B.D.+47 2291 and No. 367 in Birmingham's Catalogue of Red Stars (1877). Position for 1900: R.A. $15^h\ 59^m\ 43^s$ Dec. $+47^\circ\ 30'{\cdot}6$. The colour is a full orange which sometimes appears tinged with red and the spectrum Mc5d according to H.C.O.

Variability was announced by J. E. Gore of Dublin in 1890, and confirmed by Yendell (Dorchester, U.S.A.), who found a period of $94^d{\cdot}5$. Chandler in his Third Catalogue adopted Yendell's period and range from magnitude 5·9 to 7·2, but later observations by Markwick and members of the Variable Star Section of the British Astronomical Association did not support the published elements or show definite variation.

Summing up results in the fifth Report of the section (1903) Col. Markwick arrives at conclusions substantially as follows: (*a*) variation not proven; (*b*) the observations do not confirm the period of 92·5 (*sic*) days; (*c*) if there is any variation it is so small as to be masked by the effect of the changing position angle of the asterism; (*d*) the apparent difference in brightness between the comparison stars *a* and *b* is greatest in Feb.–Apr. and falls to a minimum in Oct.–Nov. of each year; (*e*) further investigation must decide the question whether X is variable or not. In the sixth Report (1906) curves are given showing the apparent light interval between *a* and *b* from 1890 to 1905, the object being to reveal possible variation in the comparison stars. The seasonal effect above referred to is well brought out by these curves and amounts to about $0^m{\cdot}6$ (from $0^m{\cdot}4$ to $1^m{\cdot}0$). Concerning X, it is remarked that the variation remains obscure and difficult to establish from the observations.

Meanwhile the Provisional Catalogue of Harvard (1903) had already classed X Herculis as "Irregular" which the Second Catalogue (1907) changed to "Suspected."

With these antecedents in view, regular observations by the method of Argelander were commenced at Zaragoza in 1908. An examination of some of the first results suggested (1) that the star was certainly variable, but (2) the range was so small ($\pm 0^m{\cdot}4$ being indicated) that (3) a considerable number of careful observations would have to be made before the true character of the variation could be determined. X Herculis was consequently followed with close attention for some time. A refracting telescope of 3-in. aperture and power of 30 diameters was used, conditions being kept as uniform as possible. The observations were put away as they were made, unreduced, until about seven years' work had accumulated. The series now discussed extends from 1908 January to 1914 July and consists of 275 observations.

The Comparison Stars.—Only two comparison stars were used; the *a* and *b* of Markwick.

Des.	Position (1900).	B.D.	P.D.	Colour.
	h m ° ′		m	
a	15 58·5 +47 25	+47 2288 7·0	7·40	Pale orange.
b	16 04·0 +47 46	+47 2300 6·7	6·58	White.

The light interval *a*–*b* was not estimated directly, but can be deduced from the observations except when X was bright and compared with *b* only. The mean of 170 such determinations makes the interval 7·5 grades and the mean divergence $0^{G}\cdot6$; that is roughly $0^{m}\cdot06$. In order to investigate seasonal effect and possible variation in comparison stars, suspected by previous observers, the mean value of the interval for each month and all the years was first found and is as follows, expressed in grades of approximately $0^{m}\cdot1$, the subscript figures indicating the numbers of estimates:—

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1908 } 1914 }	$7\cdot1_{9}$	$7\cdot3_{17}$	$7\cdot2_{18}$	$7\cdot6_{17}$	$7\cdot6_{6}$	$7\cdot3_{16}$	$7\cdot4_{24}$	$7\cdot4_{16}$	$8\cdot0_{8}$	$7\cdot2_{8}$	$8\cdot0_{17}$	$7\cdot7_{14}$

These figures reveal a seasonal variation in the size of the *a*–*b* interval with minimum in the early months of the year and maximum in Sept.–Nov., coinciding approximately with Markwick's epochs for *the opposite phases*. The principal cause is, however, the same (personal equation of position) and is dependent upon hour angle; the difference in sign is due to the fact that Col. Markwick used binoculars giving an erect image and the writer an inverting telescope. In the present case the total effect is less than one grade. Broadly speaking, the interval appears $0^{m}\cdot03$ less than normal during Jan.–Mar., and $0^{m}\cdot03$ greater in Sept.–Dec. The amount, being too small to seriously influence the results, need not be further considered here.

For each year (omitting 1914, which has only 4 observations of this kind) the mean value of the *a*–*b* interval was found to be as follows:—

$1908=7\cdot7_{11}$: $1909=7\cdot7_{57}$: $1910=7\cdot5_{37}$: $1911=7\cdot1_{36}$: $1912=7\cdot5_{13}$: $1913=7\cdot6_{11}$.

The largest yearly departure from the general mean is approximately $0^{m}\cdot04$.

Subtracting the general mean from the means of the separate months the residuals shown in Table I. were obtained. The unit here is the tenth of a grade, which may be taken for the present purpose as $0^{m}\cdot01$.

TABLE I.

	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1908	...	-5_{1}	...	...	...	...	-5_{2}	...	$+5_{1}$	...	...	$+3_{3}$
1909	...	-1_{7}	$+3_{7}$	$+2_{6}$	$+10_{2}$	$+2_{5}$	0_{3}	$+2_{7}$	$+2_{4}$	-2_{3}	$+6_{7}$	-3_{6}
1910	-2_{3}	-1_{4}	-4_{5}	$+1_{6}$	-5_{4}	-5_{2}	-1_{5}	-2_{3}	...	-2_{3}	$+8_{3}$	$+6_{3}$
1911	-5_{4}	-5_{5}	-15_{4}	0_{4}	-2_{3}	0	-1_{4}	-5_{4}	-2_{2}	...	-5_{1}	...
1912	...	...	...	...	...	-10_{2}	-1_{5}	0_{2}	$+25_{1}$	$+10_{1}$	$+5_{2}$	$+5_{2}$
1913	-15_{1}	...	$+5_{2}$	$+5_{1}$	...	0_{2}	$+5_{3}$	...	...	-5_{1}	$+5_{1}$	...
1914	...	...	...	-5_{2}	...	-5_{1}	-15_{1}	...	...	...	...	...

March 1911, with 4 observations, shows a deficiency of 15 units ($1\frac{1}{2}$ grades), five other months with only 1 observation, and one month with 2 observations show departures of 10 units or more, all the remainder differ by less than 10 units from the mean. These figures are fairly conclusive evidence against variation in the comparison stars unless of

very short period or Algol type. Finally, a cursory examination of all the separate estimates does not support the idea of short period variation. As an example, observations made on 1909 February 3, 10, 14, 17, 22, 23, 27, March 7, 11, 12, 16, 20, 24, 30, April 2, 9, 12, 15, 16, 20 give the following 20 values for the interval *a–b* (in grades):—7½, 8, 7, 7, 8, 7, 7½, 8, 8½, 8, 8, 7½, 7½, 7, 9, 7, 7½, 8, 7, 8. The observations having been made with the object of determining the magnitude of X and not the *a–b* interval, a certain fluctuation in the deduced value of the latter is to be expected, apart from errors, through the accommodation of the estimates to fit the two intervals into whole grades. There is no evidence of variability in either of the comparison stars.

Moonlight.—Since the colour of one of the stars compared (*b*) differs considerably from that of the other two, moonlight might be expected to have some influence on the estimates. Table II. shows the mean value of the interval *a–b* as estimated (indirectly) on dark nights and in the presence of moonlight of three different intensities. The two last columns give the means of the individual divergences and the number of observations respectively.

Table II.

Field.	*a–b*.	Mean Divergence.	N.
	G.	G.	
Dark sky	7·4	0·55	92
Slight moonlight	7·8	0·70	21
Moderate moonlight	7·5	0·70	32
Strong moonlight	7·7	0·54	29
	7·5	0·60	165

Moderate or strong moonlight appears to increase the apparent interval, though the quantity found is only about two-hundredths of a magnitude, but on account of its deeper colour, the effect with X may be somewhat greater. The figures in the third column indicate that the accidental errors are unaffected by moonlight.

Interpretation of the Observations—Light-curve.—Taking comparison star *a* as zero, the brightness of X has been deduced in grades by a slightly modified form of the method of Argelander. It is not possible to find precise equivalents in star magnitudes, as, since only two comparison stars are involved, the probable errors of the photometer determinations are too large to enable the exact value of the grade to be obtained. There is reason to suppose, however, that the grade is very nearly $0^{m}\cdot1$, so that the approximate magnitudes marked on the light-curve diagram are probably near the truth.

Each dot on the diagram represents a single observation. When the irregularities are very small, relative to the unit employed in estimation, a smooth curve has been drawn among the slightly scattered dots. The same has been done where the observations are too few to indicate the nature of possible short term variations; otherwise the curve has been drawn as far as possible through all the full weight dots. No

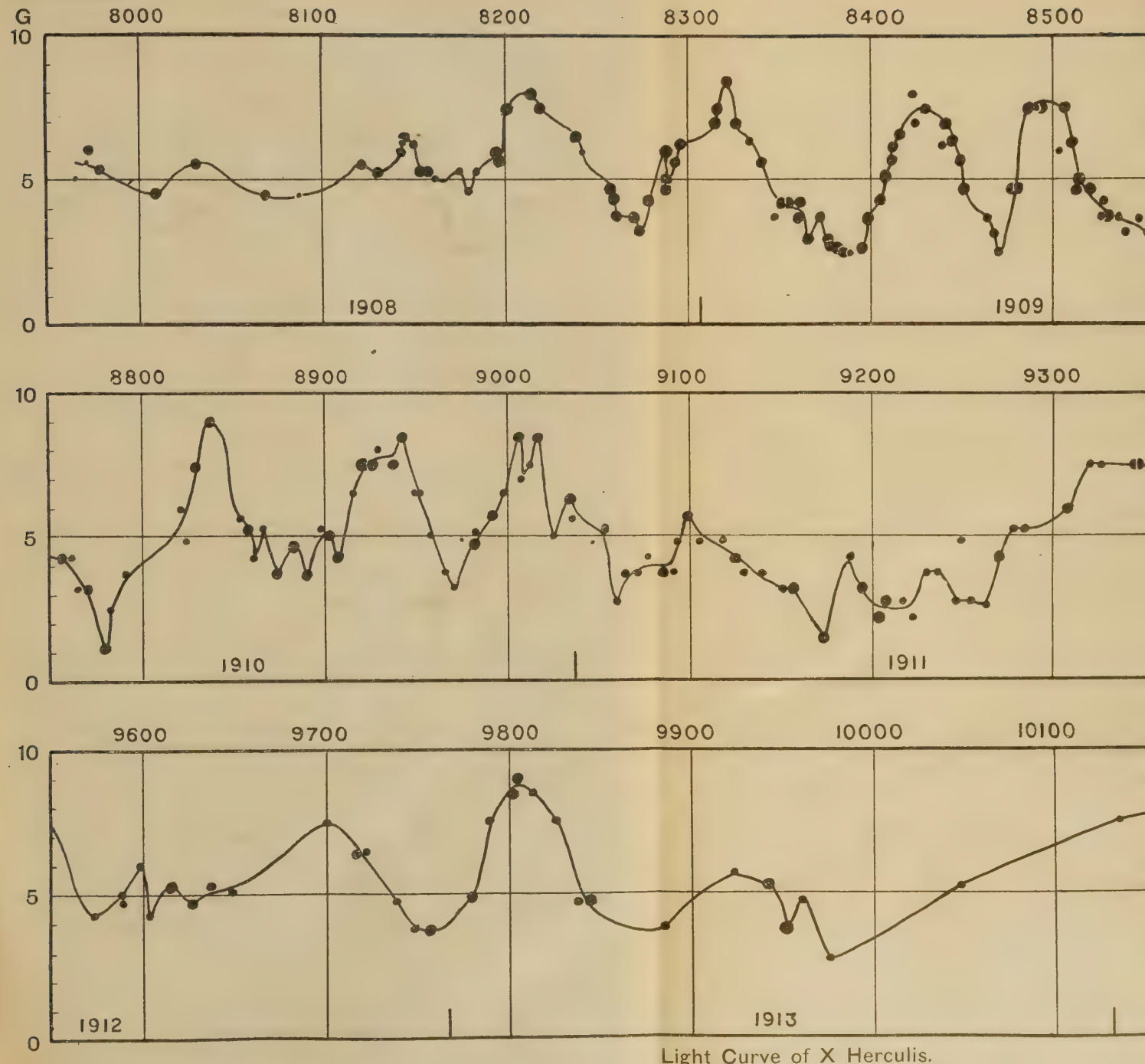
doubt it would have been more usual to have drawn a smooth curve among the moderately scattered dots also, but such smoothing entails the *à priori* assumption that corresponding short term fluctuations in the light of the star are improbable, and we have no information or experience on which to base such an assumption. At the same time, the course taken does not necessarily imply that minor sinuosities in the curve, well within the possible errors of observation, are believed to represent real phenomena. The curve is drawn through the points representing the observations because there is no reason to draw it through anywhere else.

The recurrence of identical forms and similarity to the more definitely established light-curves of other stars may be taken as evidence of reality. In this connection it is interesting to note that some of the minor irregularities in the accompanying curve are very similar to those which, on a larger scale, can be recognised in the light-curves of certain other variable stars, including R Coronæ and some *novæ*.

It will be seen that X Herculis is an irregular variable of an interesting type. I do not know of an exact counterpart, but it is probable that such could be found amongst stars of spectral types between Mc and Md, many of which appear to vary irregularly and within narrow limits. During the period covered by the present series X Herculis did not exceed the limits of one magnitude, but a few observations falling without that period show that the star was unusually bright at the end of 1906 and beginning of 1907, and again in the autumn of 1923; so the extreme amplitude of the variation is probably very near the $1^{m}\cdot3$ previously found. It is now easy to see how the period of $94^{d}\cdot5$ came to be announced; such a conclusion would easily be drawn from a short or incomplete series of observations obtained at a time when the star was behaving as in 1909 and 1910, and there is no doubt that the early observations of Yendell, which revealed this variation, were of very good quality. The period of about three months, though it keeps cropping up, is, however, not persistent, and there are times when the variation is quite irregular or the light nearly constant for several months. There is a longer term of about eighteen months, but like the three months' period it is not regular.

In exhibiting a rather unsuccessful tendency to conform to a regular period X Herculis resembles R Scuti, but does not suffer the sudden drops to Algol-like minima characteristic of that star and some other irregular variables. RX Camelopardalis presents a closer analogy, but during the time that it was being studied by Martin and Plummer (*M.N.*, 76, 7), the variation was more continuous and the periodic tendency more marked. Curiously enough, the mean period of RX Camelopardalis (*i.e.* the mean interval between the successive maxima shown by Martin and Plummer) is just 94 days.

Zaragoza:
1924 *January* 7.



Light Curve of X Herculis.

Telescopic Triple Object-Glass of High Relative Aperture.

By Lt.-Col. J. W. Gifford.

A paper on a similar subject was read by the author and communicated to the Society by the late Sir David Gill in Dec. 1908. This paper was published in the *Monthly Notices of the Royal Astronomical Society*, 69, No. 2, at approximately the same time. The object-glass then described had an aperture of 3 inches only, with a focal length of 37·5 inches, the ratio of aperture to focus thus being 1 to 12·5.

The glasses which were used were:—

	ν.	μ_D.
(1) Schott's Baryta light flint with	50·3	1·57433.*
(2) ,, Borosilicate flint with	45·8	1·57574.
(3) Mantois' Borosilicate crown with	60·7	1·50112.

A few years later a somewhat similar triple of 5 inches aperture, and the same relative aperture was computed and constructed, and is now in use at Chard. It was found that the Mantois' Borosilicate crown was somewhat hygroscopic and had occasionally to be wiped after extreme damp weather.

In place therefore of this crown, both for the reason above named and for the sake of experiment, Mantois' heavy Barium crown of ν 58·6 and a refractive index of 1·61158 for the mean D line was substituted for the Borosilicate crown already referred to.

Tested by the proportional spectra, a method, however, of which the results are liable to be more or less upset by thickness, a slight improvement in the residual tertiary spectrum should have been effected. This glass was soft and liable to be scratched, but was not hygroscopic, nor has it since suffered appreciably from surface efflorescence. In this 5-inch objective the second and third surfaces were united by cement (balsam).

The late Sir David Gill saw and approved of this objective when under test before the final operations had been carried out. He considered it would be a success. Unhappily, his unexpected death in 1913 put an end to his valued help and prevented this opinion being confirmed by him.

This instrument was not completed until after the war, but was then found to be as he had predicted, the star image closely resembling that of the 3 inch in merit. However, before a year had elapsed it was clear that for some cause then unknown a very pronounced marginal astigmatism had set in quite ruining the star image. It was found that no amount of readjustment of the lenses with regard to each other, or of squaring on, would remove this. Finally the balsam was entirely removed and a radical cure thus effected. The loss of light thus occasioned is small and the freedom from residual colour considerable.

With a power of approximately 180 obtained by a Zeiss 9 mm. orthoscopic eyepiece on a watch dial at suitable distance, the margin of the dial just shows a faint dull brick-coloured fringe. At the same

* For fuller details see Appendix.

time as the 5 inch another triple had been laid down for construction, having an aperture of $4\frac{1}{16}$ inch, and a focal length of 37·5 inch, the same as that of the original 3 inch. Thus the relative aperture was now $f/9{\cdot}2$ instead of $f/12{\cdot}5$, an aperture it is believed not previously obtained with an apochromatic triple of these dimensions. The glasses used were :—

	ν.	μ_D.
(1) Schott's Borosilicate crown	63·7	1·51229.*
(2) „ Telescope flint	51·5	1·52944.
(3) Mantois' Baryta light „	55·5	1·56797.

Instead of equalising focal lengths for B′, A, and Φ (wave-lengths 7066, 5608, and 4678 respectively), as originally recommended (*loc. cit.*), in this case B′ and the hydrogen line G′ (wave-length 4341) were used.

In the concluding stages it was found that this triple was markedly undercorrected. Focal lengths for B′, A, and Φ were then measured, and the lens recomputed for just as much overcorrection as the undercorrection found.

This was effected in the combination by slightly lengthening the last radius of curvature. Everything then worked out right, and the result was pronounced very excellent by two well-known and efficient critics in these matters. The freedom from colour and the brilliancy of the star image achieved by this objective is beyond ordinary description. It not only affects the spurious disc but the diffraction rings also, which are extremely thin and tenuous. I consider that the undercorrection of this $4\frac{1}{16}$ inch is not entirely due to the line G′ having been used for equalisation instead of Φ, but that the extended relative aperture has rendered the combination very sensitive to small errors in the refractive index of the melting. These three optical glasses have been included in a list of glasses (*Roy. Soc. Proc.*, A **87**, 1912 June 27), and their refractive indices and the probable difference in index to be found in the blocks from each melting there indicated. On reference it will be found that with S. 0,5077 Borosilicate crown this probable difference was $\pm 0{\cdot}00005$, with S. 0,4805 Telescope flint $\pm 0{\cdot}00013$, and with M. 4087 Baryta light flint $\pm 0{\cdot}00011$, and these are the glasses already referred to as having been used for this triple. In the last two glasses these differences are much beyond the normal, and in case of their both being in the same direction, and perhaps the crown as well, the total error from this cause alone would be as much as $\pm 0{\cdot}00029$ in the index, an amount sufficient to upset seriously the very delicate adjustment for freedom from colour in an apochromatic triple with such an aperture ratio.

Another apochromatic triple has been computed since those here described, and with the original glasses, with the exception of Mantois' somewhat hygroscopic crown, for which the crown used in the $4\frac{1}{16}$ inch has been substituted. The object-glass is in this case of $7\frac{1}{4}$ inch aperture, with a ratio of aperture to focal length as low as 16·0. The glass discs for this objective are ready and have been successfully examined under crossed

* For fuller details see Appendix.

nicols. Although quite ready for the grinder, in the present state of things in this country the money for its construction has not yet been forthcoming.

The grinding and polishing of both those here described, and which have been constructed, has been carried out with much skill by Mr. Underhill of the firm of Messrs. Hilger, Ltd. I have again to thank Mr. Nelson, who has done so much for microscopists, and on this occasion also Mr. Merlin of the B.A.A., for their most kind and valuable help in criticising both of the instruments here described.

Fuller details of the object-glasses having been called for, an Appendix has been added.

APPENDIX.*

Measurements in Inches.

	No. 1.	F = 37·5 in.	Effective Aperture = 3·0 in.		
lasses :	Schott's 0,3187	$R_0 = 18·281$ in.	$t_1 = 0·450$ in.	$R_2 = -5·5445$ in.	$t_3 = 0·001$ in.
	,, 0,364	$R_4 = R_2$	$t_5 = 0·150$ in.	$R_6 = 9·1391$ in.	$t_7 = 0·003$ in.
	Mantois' 3970	$R_8 = R_6$	$t_9 = 0·350$ in.	$R_{10} = -138·27$ in.	
	No. 2.	F = 62·5 in.	Effective Aperture = 5·0 in.		
lasses :	Schott's 0,3187	$R_0 = 35·372$ in.	$t_1 = 0·750$ in.	$R_2 = -7·7975$ in.	$t_3 = 0·001$ in.
	,, 0,364	$R_4 = R_2$	$t_5 = 0·250$ in.	$R_6 = 16·454$ in.	$t_7 = 0·006$ in.
	Mantois' 4078	$R_8 = 16·732$	$t_9 = 0·550$ in.	$R_{10} = 373·33$ in.	
	No. 3.	F = 37·5 in.	Effective Aperture = 4·0625 in.		
lasses :	Schott's 0,5077	$R_0 = 30·745$ in.	$t_1 = 0·563$ in.	$R_2 = -6·3723$ in.	$t_3 = 0·000$ in.
	,, 0,4805	$R_4 = R_2$	$t_5 = 0·1875$ in.	$R_6 = 6·3723$ in.	$t_7 = 0·0038$ in.
	Mantois' 4087	$R_8 = 6·4223$ in.	$t_9 = 0·5625$ in.	$R_{10} = -96·175$ in.	
	No. 4.	F = 116·0 in.	Effective Aperture = 7·25 in.		
lasses :	Schott's 0,3187	$R_0 = 37·407$ in.	$t_1 = 1·125$ in.	$R_2 = 10·719$ in.	$t_3 = 0·000$ in.
	,, 0,3193	$R_4 = R_2$	$t_5 = 0·325$ in.	$R_6 = 27·261$ in.	$t_7 = 0·100$ in.
	,, 0,5077	$R_8 = 37·615$ in.	$t_9 = 0·625$ in.	$R_{10} = \infty$.	

* R stands for the different radii of spherical curvature, t for the thicknesses, F for the equivalent foci. For complete details of the glasses, their optical constants, and a list of refractive indices for each, see *Roy. Soc. Proc.*, 87, p. 189.

The Meteoric Showers of January and December (Quadrantids and Geminids). By W. F. Denning.

An important feature in the observation of a meteoric shower is the determination of the time of maximum abundance. This is not so easy a matter as might be considered on account of the different conditions affecting the visibility of a shower from year to year. With care and repeated efforts, however, the feature alluded to may be ascertained with considerable certainty, and this is very desirable, for it enables regular or casual observers to be notified as to the exact time when they must look out in order to witness the most important and attractive phase of a periodical meteoric display.

I believe the Quadrantids of January ($230° + 52°$) and the Geminids of December ($112° + 33°$) have in late years given very distinct evidence of the exact time of their individual maxima.

The January meteors in 1906 were very rich soon after sunset on January 3, when several fine objects from this stream were seen in the evening twilight by the Rev. T. E. R. Phillips and myself while we were observing Jupiter at Bristol through my $12\frac{1}{2}$ and 10-inch reflecting telescopes. An unusually numerous shower was repeated in 1918 January 3 about 6 p.m. and in 1922 at a similar date and hour. There were a large number of Quadrantid Meteors recorded in 1838 by Wartman at Geneva, and in 1848 by M. Colla and E. Heis. In 1864 the hourly number reached three a minute, and the display was favourably observed by Professor A. H. Herschel, R. P. Greg, E. J. Lowe, and other observers, all of whom testified to the striking activity of the event.

The Sun's longitude at the times of occurrence of the two latter showers and of some more recent displays is tabulated below for comparison :—

		Sun's Longitude.	
1848 Jan.	2	281° 52′	Herschel, Greg, and Lowe.
1864	2	281 56	Heis and Colla.
1887	3	281 54	T. W. Backhouse.
1906	3	282 29	Rev. T. E. R. Phillips and W. F. D.
1918	3	282 34	Mrs. Fiammetta Wilson and W. F. D.
1922	3	282 36	W. F. D.

According to this evidence we may expect rich returns as under :—

Quadrantids.

Year.		Civil Time.
		h
1925 Jan.	3	12
1926	3	18
1927	3	24
1928	4	6
1929	3	12
1930	3	18
1931	3	24

The December meteors were seen in considerable numbers on 1920 December 12, 14^h (Sun's long. = 260° 47′). They were also very active on 1923 December 13 in early evening. Future maxima will probably occur as follows:—

Geminids.

Year.		Civil Time.
		h
1924	Dec. 12	14
1925	12	20
1926	13	2
1927	13	8
1928	12	14
1929	12	20

It will be important to watch the displays at the given times and determine whether these computations accord with observation.

The Perseids furnished a splendid maximum of 250 per hour on 1921 August 12, $2\frac{1}{2}^h$ civil time (Sun's long. = 138° 51′). By comparing a great number of rich returns recorded during the last twelve centuries I found a period of 11·72 years (*Monthly Notices*, 1923 November). This indicates future max. (if the basis adopted is reliable for the future outlook) at about:—

1932·88	1956·32
1944·60	1968·04

The period is somewhat uncertain. It requires careful reinvestigation.

Moonlight sometimes affects the character of the displays, or at places where the principal observers reside the max. may return in daylight. In other years cloudy weather may partly or wholly obliterate a shower.

Observations at different places and by different persons are not strictly comparable. According to the ability of observers and the favourableness of a station and of the atmosphere so the results must be influenced. All these circumstances naturally make a miscellaneous collection of results very difficult to compare with perfect fairness, and before a good judgment can be founded on such data we must know the details of the observations, be able to estimate their relative values, and make allowances accordingly.

Bristol:
1924 *January* 4.

Détermination de l'orbite de la comète 1922*c d'après trois observations.* Par B. Noumerov.

(*Communicated by Sir Frank Dyson.*)

L'article proposé donne une exposition élémentaire de la nouvelle méthode pour la détermination des orbites d'après trois observations et son application à la comète de 1922, observée le 23, 24, et 25 octobre.

La détermination d'une orbite, ou mieux vaut dire des coordonnées héliocentriques de l'astre, correspondantes aux trois observations, doit absolument satisfaire à deux conditions, dont la première, celle de l'observation, peut être exprimée ainsi :

Dans le cas $\lvert \tan\alpha \rvert < 1$:	Dans le cas $\lvert \tan\alpha \rvert > 1$:
$y = mx + a$	$x = my + a$
$z = nx + b$	$z = ny + b$

. . . (1)

où nous avons mis pour abréger :

$m = \tan\alpha$	$m = \cot\alpha$
$n = \sec\alpha \tan\delta$	$n = \operatorname{cosec}\alpha \tan\delta$
$a = m\mathrm{X.} - \mathrm{Y.}$	$a = m\mathrm{Y.} - \mathrm{X.}$
$b = n\mathrm{X.} - \mathrm{Z.}$	$b = n\mathrm{Y.} - \mathrm{Z.}$

. . (2)

Dans ces dernières formules α et δ sont l'ascension droite et la déclinaison de la direction observée, X., Y., Z.—les coordonnées du Soleil et x, y, z—les coordonnées héliocentriques à trouver.

Les formules (1) et (2) résultent directement de celles du changement des coordonnées héliocentriques en coordonnées géocentriques, par suite de l'élimination de la distance géocentrique.

Il suit de (1), que les trois coordonnées inconnues x, y, z pour chaque observation sont liées par deux rélations linéaires.

La seconde condition, celle du mouvement d'après la loi Newtonienne, peut être exprimée a l'aide d'une équation entre trois valeurs successives de quelque coordonnée héliocentrique, par exemple x_1, x_2, x_3 ; cette dernière peut être obtenue d'une façon élémentaire en partant des considérations suivantes ; x_1, y_1, z_1, x_2, y_2, z_2 étant les coordonnées correspondantes aux moments t_1 et t_2, la coordonnée x_3 pour un moment troisième sera :

$$x_3 = x_2 + (x_2 - x_1)\tau + \frac{\tau(\tau - 1)}{2}\Delta^2 \quad . \quad . \quad . \quad (3)$$

où

$$\tau = \frac{t_3 - t_2}{t_2 - t_1} = \frac{\omega'}{\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

et Δ^2 est la seconde différence de la fonction x pour le moment t_2.

L'équation différentielle du mouvement :

$$\frac{d^2x}{dt^2} = -\frac{k^2 x}{r^3} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

donne :

$$\Delta^2 = (t_2 - t_1)^2 \frac{d^2x}{dt^2} = -\frac{k^2\omega^2 x}{r^3} \quad . \quad . \quad . \quad (6)$$

de façon, que l'équation (3) devient définitivement :

$$x_1 \frac{\tau}{1+\tau} + x_3 \frac{1}{1+\tau} = x_2(1-\lambda_2) \quad . \quad . \quad . \quad (7)$$

où

$$\lambda_2 = \frac{k^2\omega\omega'}{2r_2^3} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

L'équation (7) est précise jusqu'au second ordre; cette précision suffit en général dans le cas des observations séparées par petits intervalles, ainsi que pour les comètes ayant une grande distance périhélie.

Gibbs proposa une rélation entre trois valeurs successives des coordonnées, précise jusqu'au quatrième ordre (Bauschinger, *Bahnbestimmungen*, p. 179), qui a été ensuite généralisée par moi (Voir : "Méthode nouvelle pour la détermination des orbites et le calcul des éphémérides en tenant compte des perturbations." *Publications de l'Observatoire Astrophysique à Moscou*, vol. ii.).

Donc, les conditions de l'observation et du mouvement, suivant la loi de Newton, devant être absolument satisfaites, la détermination des orbites répose sur les rélations (1) et (7).

En considerant ensemble l'équation (1), écrite pour toutes les observations et (7), pour toutes les coordonnées x, y, z, nous obtenons facilement les formules suivantes, pour la détermination des coordonnées héliocentriques :

$$\left.\begin{aligned} x_1 \frac{\tau}{1+\tau} &= F(0,1) + \lambda_2 F(2,1) \\ x_2(1-\lambda_2) &= F(0,2) + \lambda_2 F(2,2) \\ x_3 \frac{1}{1+\tau} &= F(0,3) + \lambda_2 F(2,3) \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

où

$$\left.\begin{aligned} D\mu_1 &= m_2 - m_3 & D\nu_1 &= n_2 - n_3 \\ D\mu_2 &= m_1 - m_3 & D\nu_2 &= n_1 - n_3 \\ D\mu_3 &= m_1 - m_2 & D\nu_3 &= n_1 - n_2 \end{aligned}\right\} \quad . \quad . \quad (10)$$

$$D = (m_1 - m_3)(n_2 - n_3) - (m_2 - m_3)(n_1 - n_3) = \begin{vmatrix} 1 & 1 & 1 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{vmatrix} \quad (11)$$

$$\left.\begin{aligned} a_0 &= a_1 \frac{\tau}{1+\tau} + a_3 \frac{1}{1+\tau} - a_2 \\ b_0 &= b_1 \frac{\tau}{1+\tau} + b_3 \frac{1}{1+\tau} - b_2 \end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

$$F(i,j) = b_i\mu_j - a_i\nu_j \quad . \quad . \quad . \quad . \quad (13)$$

Le facteur λ_2 étant inconnu, la seconde équation du système (9) ne peut être resolue que par des approximations successives : on fixe une valeur pour x_2 et trouve y_2 et z_2 d'après la formule (1) ; l'équation (8) nous donne λ_2 et enfin le système (9) permet de trouver x_2. Ce système analogue à l'équation de Gauss peut donner des racines multiples.

En titre d'exemple * prenons la détermination des coordonnées héliocentriques équatoriales réctangulaires de la comète de 1922*c* d'après les observations suivantes :

TABLE I.

1922.	G.M.T.	$\alpha_{app.}$	$\delta_{app.}$	$\Delta\alpha$.	$\Delta\delta$.	Observatoire.
		h m s	° ′ ″	s	″	
I.	Oct. 23·28035	19 55 2·25	+36 41 32·0	+1·83	+32·5	Copenhague
II.	,, 24·27500	19 57 15·82	36 24 23·1	1·83	+32·5	Copenhague
III.	,, 25·29666	19 59 33·46	36 6 44·6	1·84	+32·5	Pulkovo

Les coordonnées du Soleil, corrigées pour la parallaxe par la réduction au lieu de l'observation, suivant les formules :

$$\Delta x = -\rho \cos\phi \cos S$$
$$\Delta y = -\rho \cos\phi \sin S$$
$$\Delta z = -\rho \sin\phi$$

où S est le temps sidéral local, doivent être interpolées au moment de l'observation :

TABLE II.

Moments.	X	$10^6\,\Delta x$.	Y.	$10^6\,\Delta y$.	Z.	$10^6\,\Delta z$.
Oct. 23·28035	−0·865015	−20	−0·450320	+14	−0·195328	−35
24·27500	−0·856153	−20	−0·463855	+14	−0·201199	−35
25·29666	−0·846787	−21	−0·477614	+4	−0·207166	−37

Les formules (2) nous donnent :

TABLE III.

	I.	II.	III.
α_{1922}	$19^h\ 55^m\ 0^s{\cdot}42$	$19^h\ 57^m\ 13^s{\cdot}99$	$19^h\ 59^m\ 31^s{\cdot}62$
δ_{1922}	36° 40′ 59″·5	30° 23′ 50″·6	36° 6′ 12″·1
log tan δ	9·872110	9·867582	9·862907
log cosec α	0·057143*n*	0·059484*n*	0·061953*n*
log cot α	9·739303*n*	9·749244*n*	9·759367*n*
log Y.	9·653508*n*	9·666369*n*	9·679073*n*
log *n*	9·929253*n*	9·927066*n*	9·924860*n*
*m*Y.	0·247066	0·260383	0·274435
−X.	0·865035	0·856173	0·846808
a	1·112101	1·116556	1·121243
*n*Y.	0·382614	0·392135	0·401729
−Z.	0·195363	0·201234	0·207203
b	0·577977	0·593369	0·608932

Ensuite trouvons d'après les formules (10), (11), (4), et (12) :

* Le calcul a été fait par les collaborateurs de l'Institut des Calculs de Pétrograd.

Table IV.

−0·548660	$D\mu_1$	+0·013237	μ_1	+ 6388	$D\mu_2\nu_1$	−0·000111131	ω'	1·02166
−0·561364	$D\mu_2$	+0·025941	μ_2	12519	$-D\mu_1\nu_2$	+0·000113203	ω	0·99462
−0·574601	$D\mu_3$	+0·012704	μ_3	6131	D	0·000002072	τ	1·027186
							$\tau:(1+\tau)$	0·506705
−0·849676	$D\nu_1$	−0·004284	ν_1	− 2068	D^{-1}	4826	$1:(1+\tau)$	0·493295
−0·845408	$D\nu_2$	−0·008552	ν_2	− 4127			a_0	+0·000055
−0·841124	$D\nu_3$	−0·004268	ν_3	− 2060			b_0	−0·000122

Enfin les formules (13) et (9) nous permettent de trouver :

Table V.

i.	I.	II.	III.
$b_2\mu_i$	3790	7429	3638
$-a_2\nu_i$	2309	4608	2300
F(2, i)	6099	12037	5938
$b_0\mu_i$	−0·7794	−1·5274	−0·7480
$-a_0\nu_i$	+0·1137	+0·2270	+0·1133
F(0, i)	−0·6657	−1·3004	−0·6347
λ_2F(2, i)	+0·0466	+0·0919	+0·0454
F(0, i)+λ_2F(2, i)	−0·6191	−1·2085	−0·5893
y_i	−1·2218	−1·2085	−1·1946
x_i	1·7825	1·7950	1·8077
z_i	1·6161	1·6150	1·6137

La dernière approximation dans la résolution de la seconde équation du système (9)

$$y_2 = \mathrm{F}(0, 2) + \frac{k^2\omega\omega'}{2}\left[\mathrm{F}(0, 2) + \mathrm{F}(2, 2)\right]\frac{\mathrm{I}}{r_2^3} = \mathrm{F}(0, 2) + \frac{1}{t}\left[0{\cdot}25758\right]\frac{\mathrm{I}}{r_2^3} \quad (9)^*$$

sera :

Table VI.

y_2	−1·2084
x_2	+1·7949
z_2	1·6150
r_2^2	7·2901
F(0, 2)	−1·3004
y_2-F(0, 2)	+0·0919
y_2	−1·2085

Le calcul a été fait pour la plupart en se servant de l'arithmomètre, et bien rarement à l'aide des tables logarithmiques.

Addressons nous maintenant à la méthode de la variation des coordonnées héliocentriques, qui nous présentera un moyen de contrôler notre calcul, ainsi que d'établir la précision du résultat obtenu. Ayant adopté pour le 1^er^ et le 3^ème^ moments les valeurs: $y_1 = -1{\cdot}221800$ et $y_3 = -1{\cdot}194600$, obtenues plus haut, et dont nous avons augmenté de gré la précision jusqu'à la sixième décimale, nous avons pour x_1, z_1, x_3, z_3, d'après la condition de l'observation (1):

TABLE VII.

	x.	Δ^1.	Δ^2.	y.	Δ^1.	Δ^2.	z.	Δ^1.	Δ^2.
t_1 23·28035	1·782454			−1·221800			1·616111		
		+25207			+27200			−2372	
t_3 25·29666	1·807661		−110	−1·194600		+73	1·613739		−98

$$\frac{r^2}{k^2\omega^2/r^3}.$$

7·298861

0·000061008

Le système des coordonnées pour ces deux moments répond précisement à la première et troisième observation. Reste à calculer la seconde différence des fonctions x, y, z pour le troisième moment (6) et d'interpoler les coordonnées pour le moment $t_2 = 24{\cdot}27500$ jusqu'aux secondes différences. En comparant les valeurs obtenues aux conditions de l'observation (1) nous aurons:

TABLE VIII.

x_2.	y_2.	z_2.
1·794902	−1·208392	1·614953
1·794904		1·614953
$\beta = -2$		$\gamma = 0$

où nous avons mis:

$$\left.\begin{aligned}\beta &= x_2 - (m_2 y_2 + a_2)\\ \gamma &= z_2 - (n_2 y_2 + b_2)\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

de façon que β et γ expriment la différence entre les coordonnées de la comète calculées pour le moment t_2 et les données de la seconde observation.

Dans le cas présent β et γ ne sont pas grandes, parce que nous nous servons des valeurs y_1 et y_3 déjà assez précises, mais en général ces valeurs β et γ doivent servir à la correction des coordonnées y_1 et y_3.

Les formules différentielles pour la correction des coordonnées fondamentales résultent de la différentiation de (7), en négligeant le changement de λ_2:

$$\left.\begin{aligned}\tau\Delta y_3 &= \gamma\mu_3 - \beta\nu_3\\ (1-\tau)\Delta y_1 &= \gamma\mu_1 - \beta\nu_1\end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

où nous avons mis

$$\tau = \frac{t_2 - t_1}{t_3 - t_1}.$$

Pour notre exemple les formules (15) seront :

$$\left.\begin{aligned}\Delta y_1 &= 12608\gamma + 4080\beta \\ \Delta y_3 &= 12429\gamma + 4176\beta\end{aligned}\right\} \quad . \quad . \quad . \quad (15)^*$$

Cette dernière formule montre, que dans le cas présent les coordonnées ne peuvent être déterminées que d'une façon bien peu exacte ; supposons que la précision des valeurs β et γ est celle du calcul, c'est à dire 10^{-6}, nous aurons pour l'erreur des coordonnées $E_1 = \pm 0{\cdot}013$.

Enfin il suit des formules (14), (1) et (2)

$$\left.\begin{aligned}\Delta\beta &= \rho \frac{\cos\delta}{\sin\alpha}\Delta\alpha \\ \Delta\gamma &= -\rho \sec\delta\Delta\delta + \rho \sin\delta \cot\alpha\Delta\alpha\end{aligned}\right\} \quad . \quad . \quad (16)$$

ou pour notre cas :

$$\left.\begin{aligned}\Delta\beta &= -2{\cdot}2\Delta\alpha \\ \Delta\gamma &= -2{\cdot}9\Delta\delta - 0{\cdot}8\Delta\alpha\end{aligned}\right\} \quad . \quad . \quad . \quad (16)^*$$

Les formules (15)* et (16)* permettent de conclure, que la précision de $\Delta\alpha$ et $\Delta\delta$ étant 1″, nous aurons pour les coordonnées y_1 et y_2 une erreur proche à $E = \pm 0{\cdot}2$.

Ce résultat montre que pour la première détermination de l'orbite la précision des coordonnées reçues et, de suite des éléments, est bien petite.

L'examen de la Table VII montre que c'est la condition de l'observation et non celle du mouvement d'après la loi Newtonienne, qui est d'importance dominante pour toute la détermination de l'orbite.

Le mouvement Newtonien se traduit ici dans la présence des secondes différences, mais leur influence dans le procès d'interpolation étant peu sensible, on peut les mépriser totalement, ce qui sera égal au cas du mouvement uniforme réctiligne. Donc nous aurons :

Table IX.

x.	y.	z.
1·794888	−1·208383	1·614941
1·794899		1·614946
$\beta = -11$		$\gamma = -5$

La formule (15)* nous donnera $\Delta y_1 = -0{\cdot}108$, $\Delta y_3 = -0{\cdot}108$.

Cela veut dire que les différences des coordonnées pour le cas du mouvement réctiligne ainsi que pour celui de la loi Newtonienne, seront comprises dans les mêmes limites que les erreurs mêmes de l'observation.

Le calcul des éphémérides pour de petites intervalles de temps est fait le plus simplement d'après la méthode de Cowell et Crommelin ; tandis que dans le cas de grandes intervalles, il est commode d'introduire les coordonnées spéciales, se rapportant aux ordinaires par l'équation :

$$\bar{x} = x\left(1 + \frac{k^2\omega^2}{12r^3}\right), \quad \bar{y} = y\left(1 + \frac{k^2\omega^2}{12r^3}\right), \quad \bar{z} = z\left(1 + \frac{k^2\omega^2}{12r^3}\right) \quad . \quad (17)$$

Alors les coordonnées pour le troisième moment seront calculées d'après les coordonnées des deux premiers moments suivant la formule :

$$\bar{x}_3 = \bar{x}_2(2-\sigma_2) - \bar{x}_1 + R_2 + \tfrac{1}{12}\Delta^2 R_2 + \rho \quad . \quad . \quad (18)$$

où R_2 est l'influence des planètes dans le mouvement troublé, et le facteur σ satisfait à l'équation :

$$\sigma\left(1-\frac{\sigma}{12}\right)^2 = \frac{k^2\omega^2}{\bar{r}^3} \quad . \quad . \quad . \quad . \quad (19)$$

Enfin ρ est l'influence des ordres supérieurs, commençant par le sixième

$$\rho = \frac{1}{240}\Delta^6 - \frac{31}{60480}\Delta^8 \quad . \quad . \quad . \quad . \quad (20)$$

Notons que pour d'intervalles assez grandes ρ peut être négligé. Le changement des coordonnées spéciales en coordonnées ordinaires est fait suivant la formule :

$$x = \bar{x}\left(1-\frac{\sigma}{12}\right). \quad . \quad . \quad . \quad . \quad (21)$$

Le résultat définitif de notre calcul est une éphéméride de la comète pour les deux mois prochains ; le minimum de r nous permet de déterminer la distance q du périhélie au Soleil, ainsi que le temps T_0 du passage de la comète par son périhélie.

TABLE X.

Ephéméride de la comète 1922*c*.

G.M.T.		α.	δ.	log ρ.
		h m	° ′	
Oct.	23·5	19 55·5	+36 37	0·376
Nov.	2·5	20 18·2	33 41	
	12·5	20 41·3	30 54	
	22·5	21 4·4	28 12	0·415
Dec.	2·5	21 27·1	25 44	
	12·5	21 49·3	23 33	
	22·5	22 10·6	22 12	0·481

$\log q = 0{\cdot}4232 \qquad T_1 = 1922$ Sept. $23{\cdot}89$.

Pétrograd,
Observatoire astronomique
de l'Université :
1922 *Novembre* 12.

Results of Micrometer Measures of the Positions of Jupiter's Satellites relative to the Star B.D. – 14° 4045 made with the 243 mm. Merz-Repsold Refractor at the Astronomical Observatory, Kiew. By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

On 1923 May 7 clouds prevented the observation of the occultation of the star B.D. – 14° 4045 * by Jupiter. But on 1923 May 5, 6, and 8, I measured with the filar micrometer the differences of the right ascensions and declinations of Jupiter's satellites and this star B.D. – 14° 4045. The power employed was 145. The results of my micrometrical measures (the differential refraction is taken into consideration) are as follows :—

1923 *May* 5. *Satellite IV.*

Kiew Sid. Time.	$\alpha_s - \alpha_*$.	$\delta_s - \delta_*$.
h m s	s	"
13 8 6·2	+ 7·0	+29·36
13 13 27·8	+ 7·1	+27·62
13 18 8·2	+ 6·5	+28·30
13 26 2·7	+ 6·2	+31·21
13 31 22·6	+ 6·2	+29·16
13 34 49·9	+ 6·3	+30·55
13 38 36·2	+ 6·5	+31·32

1923 *May* 6. *Satellite II.*

12 21 10·1	– 2·5	+98·28

Satellite III.

12 21 0·3	–12·3	+98·28

1923 *May* 8. *Satellite I.*

11 58 25·9	–10·1	+31·50
12 6 46·3	– 9·8	+31·67
12 15 47·4	– 9·7	+31·48
12 30 28·6	...	+32·79
12 48 6·6	–10·61	
12 58 10·6	–10·50	
13 13 49·6	–10·93	
13 20 59·6	...	+35·21
13 26 28·6	...	+34·72
13 33 31·6	...	+37·08

* The apparent positions of the star B.D. – 14° 4045, observed with the meridian circle by M. Ditschenko, are :—

	h m s	° ′ ″
1923 June 13	14 46 51·13	– 14 42 38·90
14	51·08	38·73
16	51·03	39·37
17	51·04	39·17

Kiew Sid. Time.	$\alpha_S - \alpha_*$.	$\delta_S - \delta$.
	Satellite II.	
h m s	s	"
12 35 43·6	...	+73·28
12 52 26·6	−17·02	
12 54 16·6	−16·89	
13 16 27·6	−17·93	
13 23 15·6	...	+79·91
13 29 15·6	...	+81·25
13 37 43·6	...	+82·71
	Satellite IV.	
12 33 46·6	...	+23·11
12 51 8·6	−14·22	
12 57 5·6	−13·85	
13 15 11·6	−14·40	
13 22 10·6	...	+25·20
13 27 31·6	...	+24·74
13 36 39·6	...	+25·17

Astronomical Observatory, Kiew:
1923 *December* 19.

Occultations of Stars by the Moon, observed at the Astronomical Observatory, Kiew, in the year 1923. By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

No.	Date 1923.	Greenwich Mean Time.	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
		h m s					
1	Mar. 23	4 7 47·09	α Tauri	1·1	D	G 58	T
2	23	4 59 56·07	α Tauri	1·1	R	G 58	T
3	23	4 59 51·33	α Tauri	1·1	R	K 24	D
4	23	4 59 56·33	α Tauri	1·1	R	U 74	K
5	24	4 28 33·09	117 Tauri	6·0	D	K 24	D
6	24	5 13 28·22	117 Tauri	6·0	R	G 58	T
7	25	4 19 7·87	292 B Orionis	6·5	D	R 90	T
8	25	5 13 8·37	Anonyma	7·5	D	R 90	T
9	25	5 16 30·37	Anonyma	6·0	D	R 90	T
10	Apr. 19	7 8 16·78	θ^1 Tauri	4·2	D	G 58	T
11	19	7 8 16·30	θ^1 Tauri	4·2	D	K 24	D

No.	Date 1923.	Greenwich Mean Time. h m s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
12	19	7 8 16·16	θ^1 Tauri	4·2	D	U 74	K
13	Apr. 19	7 17 21·78	θ^2 Tauri	3·6	D	G 58	T
14	19	7 17 21·31	θ^2 Tauri	3·6	D	K 24	D
15	19	7 17 21·27	θ^2 Tauri	3·6	D	U 74	K
16	19	7 21 21·28	Anonyma	8·0	D	G 58	T
17	23	6 34 58·88	1 Cancri	6·0	D	G 58	T
18	23	6 34 57·30	1 Cancri	6·0	D	K 24	D
19	May 22	7 12 35·61	ξ Leonis	5·1	D	R 145	T
20	22	7 12 35·08	ξ Leonis	5·1	D	K 24	D
21	22	7 12 35·01	ξ Leonis	5·1	D	G 58	K
22	22	7 28 32·99	Anonyma	6·5	D	R 145	T
23	22	7 44 50·32	Anonyma	8·0	D	R 145	T
24	26	6 49 0·87	θ Virginis	4·4	D	B 61	T
25	26	6 49 1·54	θ Virginis	4·4	D	G 58	K
26	26	7 59 50·03	θ Virginis	4·4	R	G 58	K
27	Aug. 22	5 27 1·49	173 B Sagittarii	6·4	D	R 145	T
28	22	5 27 1·56	173 B Sagittarii	6·7	D	G 87	K
29	22	5 32 20·12	171 B Sagittarii	6·1	D	R 145	T
30	22	5 32 20·67	171 B Sagittarii	6·1	D	G 87	K
31	22	6 33 9·13	171 B Sagittarii	6·1	R	R 145	T
32	22	6 44 25·78	173 B Sagittarii	6·4	R	R 145	T
33	22	8 19 40·13	187 B Sagittarii	6·4	D	R 145	T
34	22	8 19 41·81	187 B Sagittarii	6·4	D	G 87	K
35	22	8 42 47·84	187 B Sagittarii	6·4	R	R 145	T
36	Sept. 20	5 54 11·10	61 B Capricorni	5·9	D	R 145	T
37	20	5 54 10·63	61 B Capricorni	5·9	D	G 68	K
38	20	7 13 49·02	61 B Capricorni	5·9	R	R 145	T
39	Oct. 27	5 41 18·77	70 Tauri	6·4	D	R 145	T
40	27	5 56 6·34	71 Tauri	4·6	D	R 145	T
41	27	6 59 4·99	θ^1 Tauri	4·2	D	R 145	T
42	27	7 0 37·74	θ^2 Tauri	3·6	D	R 145	T
43	27	7 0 38·30	θ^2 Tauri	3·6	D	G 58	K
44	27	8 2 47·53	θ^2 Tauri	3·6	R	R 145	T
45	27	8 2 47·34	θ^2 Tauri	3·6	R	G 58	K
46	27	8 5 10·67	θ^1 Tauri	4·2	R	R 145	T
47	27	8 5 10·44	θ^1 Tauri	4·2	R	G 58	K

No.	Date 1923.	Greenwich Mean Time. h m s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
48	Oct. 27	8 12 29·93	264 B Tauri	4·8	D	R 145	T
49	27	8 15 1·52	Anonyma	6·0	D	R 145	T
50	27	9 17 33·25	264 B Tauri	4·8	R	R 145	T
51	27	10 1 16·06	275 B Tauri	6·5	D	R 145	T
52	27	11 21 45·84	275 B Tauri	6·5	R	R 145	T
53	27	11 39 39·90	α Tauri	1·1	D	R 145	T
54	27	11 39 39·84	α Tauri	1·1	D	G 58	K
55	27	12 55 49·39	α Tauri	1·1	R	R 145	T
56	27	12 55 49·30	α Tauri	1·1	R	G 58	K

Instruments used :—

R = 243-mm. Merz-Repsold refractor.
K = 120-mm. Utzschneider und Fraunhofer cometseeker.
U = 72-mm. Utzschneider und Fraunhofer refractor.
B = 81-mm. Busch refractor.
G = 70-mm. Goerz refractor.

Observers :—

T = S. Tscherny.
D = M. Ditschenko.
K = A. Karamischev.

Notes.

Nos. 1, 7, 8, 9, 10, 13, 15, 17, 19, 29, 35, 44, 46, 53, 54, 55, 56 : instantaneous,
Nos. 16, 27, 32, 33, 41, 42, 45, 47, 48, 50 : good.
Nos. 36, 39, 49, 51 : satisfactory.
Nos. 2, 3, 6, 26, 31 : retardation.
Nos. 18, 25 : observed through clouds.
No. 4 : doubtful.

Astronomical Observatory, Kiew ;
1923 *December* 19.

Proper Motions of Small Stars.

(*Communicated by the Union Observatory.*)

In the *Monthly Notices*, 1923 November, p. 3, Dr. Smart writes : "With the only exception of G. C. Comstock, as far as I am aware, . . . investigators on the systematic motions of the stars have confined their attention to the brighter stars for which meridian observations are available." This overlooks the work done here exclusively on faint stars and quite independently of any meridian proper motion work.*

* The Secretaries would also draw the attention of those interested in this subject to Mount Wilson Contributions No. 168, where Mr. van Maanen discusses the motions of faint stars in the region of the Orion Nebula.

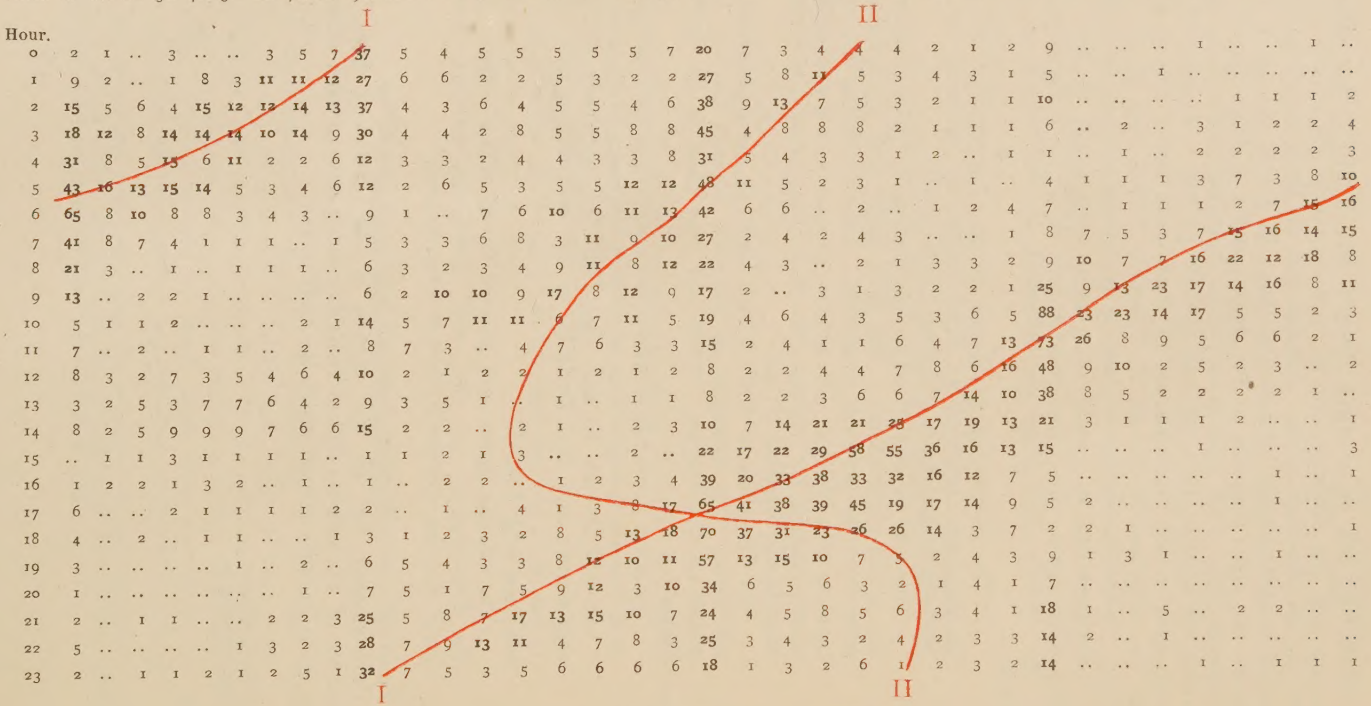
In U.O.C. No. 25 of 1915 June 2, Mr. Voûte gave a diagram showing how the displacements of faint stars near the cluster ω Cen. were in the direction of the Antapex and of Drift I.—it not being possible to differentiate between the two. In U.O.C. No. 43 of 1918 August 12, eight diagrams of the displacements of faint stars in the Greenwich Zone $+66^\circ$ were given; these diagrams have each an arrow showing the antapex drift, assumed to be $0''\cdot5$ a year. A more detailed examination of the displacements of faint stars near η Arg. is given with two diagrams by Mr. H. E. Wood in U.O.C. No. 46 of 1919 July 7. In our last circular we refer to our results derived from about 8000 displacements of nearly 6000 faint stars found on the Cape Astrographic Zone. As it may be some time before these displacements and Mr. Wood's deductions are published, his conclusions are now quoted:—

> "The distribution of the directions of motions of the stars in the Cape Astrographic Zone shows the existence of two preferential directions of motion fairly clearly.
>
> This may be seen from the following table, which summarises the actual observations. The table deals with 5702 different stars down to the 14th magnitude. Each horizontal line represents an hour of R.A. The number of stars in each hour of R.A. whose motions lay between the directions 355° and 5° is placed beneath the direction 0°, etc.
>
> The directions of Drift I. and Drift II. are represented by red lines. The direction of the solar antapex has not been placed on the diagram, but it runs very close to the Drift I. curve.
>
> Numbers greater than 9 have been printed in heavier type so as to indicate the directions of greatest frequency of motion.
>
> It will at once be seen that Drift I. is very clear and well represented, and that Drift II. is also well shown from 0^h to 12 hrs. R.A. but not from 13^h to 24 hrs. R.A., where it becomes confused with Drift I.
>
> The tabular figures are affected by an undue prevalence of the directions 0°, 90°, 180°, 270°, due to the inclusion of very small motions and the method of measurement."

These references do not exhaust our work here, but suffice.

Unfortunately for all the regions so far examined here, the directions of the antapex and of Drift I., are too close to each other to allow us to be quite certain which the stars prefer. R. I.

Johannesburg:
1924 *January* 9.



Proper Motions of Stars in the Cape Astrographic Zone.

The Total Solar Eclipse of 1922 *September* 21. (*Supplementary Note.*)

By W. E. Cooke.

In my report on the solar eclipse (*Monthly Notices*, 83, 513), I omitted to recompute the "Predicted Times," using the final adopted values for latitude and longitude of the observing station.

Mr. Merfield, in calling my attention to this omission, kindly gave me his recomputed values, viz.:

	h	m	s
1st contact .	3	05	29·7
2nd ,, .	4	11	50·4
3rd ,, .	4	15	20·4
4th ,, .	5	14	49·8

making the residuals

Obs. – Pred.
s
− 23·7
− 15·7
− 12·2
− 12·2

The above depend on the values—

Longitude of Goondiwindi Station = $10^h\ 01^m\ 12^s{\cdot}6$ E
Latitude ,, ,, ,, = $28^\circ\ 32'\ 46''{\cdot}9$ S

and the data of the *British Nautical Almanac*.